Statistics 2060 - Midterm Exam

Date: Tuesday, Mar. 4, 2008 Time: 10:05-11:25am

Name: ____________________ **Student ID #:** ____________________

This midterm exam has 3 pages with total of 30 marks. The number of points allocated to each portion of a problem is indicated on the left margin. The exam is closed notes and closed book, but one page of formula sheet (Legal sized paper with double sided writing) is permitted, and you may use a calculator. All your answers must be written on the exam papers. Partial marks can be given for partially correct answers.

1. A computer system uses passwords that are exactly seven characters and each character is one of the 26 letters (a-z) or 10 intergers (0-9). Uppercase letters are not used.

(2) (a) How many passwords are possible?

36^7

(2) (b) If a password consists of five letters followed by two numbers, how many passwords are possible?

$26^5 \times 10^2$

2. Suppose that a random variable X has the probability mass function given in the following table:

x	-1	0	1	2
$p(x)$	1/2	1/4	1/8	1/8

(2) (a) Calculate the mean of X, **E**(X).

$$EX = (-1)\times\frac{1}{2} + 0\times\frac{1}{4} + 1\times\frac{1}{8} + 2\times\frac{1}{8}$$
$$= -\frac{1}{2} + \frac{1}{8} + \frac{1}{4} = -\frac{1}{8} = -0.125$$

(2) (b) Calculate the variance of X, **V**(X).

$$EX^2 = 1\times\frac{1}{2} + 0\times\frac{1}{4} + 1\times\frac{1}{8} + 4\times\frac{1}{8} = \frac{1}{2} + \frac{1}{8} + \frac{1}{2} = \frac{9}{8}$$
$$V(X) = EX^2 - (EX)^2 = \frac{9}{8} - \frac{1}{64} = \frac{71}{64} = 1.109$$

(2) (c) Calculate $\mathbf{E}[e^X + 2X^2 + 3X]$.

$= Ee^X + 2EX^2 + 3EX$

$= e^{-1} \times \frac{1}{2} + e^0 \times \frac{1}{4} + e \times \frac{1}{8} + e^2 \times \frac{1}{8} + 2 \times \frac{9}{8} + 3 \times (-\frac{1}{8})$

$= \frac{e^{-1}}{2} + \frac{e}{8} + \frac{e^2}{8} + \frac{17}{8}$

$= 3.57$

3. Suppose that the probability density function of a random variable X is

$$f(x) = \begin{cases} 3x^2/7 & \text{if } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(2) (a) Find the cumulative distribution function of X.

for $1 \le X \le 2$

$F(x) = \int_1^x 3x^2/7 \, dx = \frac{3}{7} \cdot \frac{1}{3} x^3 \Big|_1^x = \frac{1}{7}x^3 - \frac{1}{7}$

$$F(x) = \begin{cases} 0 & , \quad x \le 1 \\ \frac{1}{7}x^3 - \frac{1}{7} & , \quad 1 \le x \le 2 \\ 1 & , \quad x > 2 \end{cases}$$

(2) (b) Find the mean of X, $\mathbf{E}$(X).

$EX = \int_1^2 x \cdot 3x^2/7 \, dx = \frac{3}{7} \cdot \frac{1}{4} x^4 \Big|_1^2$

$= \frac{3}{28}(2^4 - 1) = \frac{45}{28} = 1.607$

(2) (c) Calculate the variance of X, $\mathbf{V}$(X).

$EX^2 = \int_1^2 x^2 \cdot \frac{3x^2}{7} dx = \frac{3}{7} \frac{1}{5} x^5 \Big|_1^2 = \frac{3}{35}(2^5 - 1)$

$= 2.657$

$V(x) = EX^2 - (EX)^2 = 2.657 - (1.607)^2 = 0.0747$

(2) (d) Find the 25'th percentile of X.

$\int_1^\eta 3x^2/7 \, dx = 0.25$ or $F(\eta) = 0.25$

$\frac{1}{7}\eta^3 - \frac{1}{7} = \frac{1}{4}$ $\quad \eta^3 = 2.75$

$\eta = 1.40$.

4. Phone calls arrive at a customer service desk according to a Poisson process, with an average of 15 calls each hour.

$\lambda = 1.5$

(2) (a) What is the probability of no calls between 9:00 and 10:00 in the morning?

X: no. of calls between 9:00 and 10:00.

$X \sim pois(1.5)$

$P(X=0) = \frac{e^{-1.5}\lambda^0}{0!} = e^{-1.5} = 0.223$

(2) (b) What is the probability that there is more than one call between 9:00 and 9:10 in the morning?

X: no. of calls between 9:00 and 9:10

$X \sim pois(1.5/6)$ $\lambda = 0.25$

$P(X>1) = 1 - P(X=0) - P(X=1)$

$= 1 - \frac{e^{-0.25} 0.25^0}{0!} - \frac{e^{-0.25} 0.25^1}{1!} = 0.0265$

(2) (c) What is the probability of one call from 9:00-10:00, no calls from 10:00-11:00, and 2 calls from 11:00-12:00? (This is asking for a single probability.)

X: no. of calls between 9-10, $X \sim pois(1.5)$

Y: no. of calls between 10-11, $Y \sim pois(1.5)$

Z: no. of calls between 11-12, $Z \sim pois(1.5)$

X, Y, Z are indep.

$P(X=1 \text{ and } Y=0 \text{ and } Z=2)$

$= P(X=1) P(Y=0) P(Z=2) = \frac{e^{-1.5} 1.5^1}{1!} \cdot 0.223 \cdot \frac{e^{-1.5} 1.5^2}{2!} = 0.3347 \times 0.223 \times 0.251 = 0.0188$

5. Television sets are made on a production line and can be classified as defective or non-defective, independently of each other. Historical data indicates that the proportion of defectives produced is 5%. Let X be the number of defectives in a sample of 10 television sets.

(2) (a) What is the probability that a sample of 10 sets from the production line contains at least one defective?

X: no. of defective in 10 sets, $X \sim Bin(10, 0.05)$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - \binom{10}{0} 0.05^0 \, 0.95^{10}$

$= 1 - 0.5987 = 0.4013$

(2) (b) What is the probability that when the production line starts up in the morning, there are 20 non-defective sets produced prior to the second defective?

X: no. of non-defective prior to the second defective

$X \sim nb(x; 2, 0.05)$

$P(X=20) = \binom{20+2-1}{1} 0.05^2 \, 0.95^{20}$

$= 0.0188$

(2) (c) After the production line starts up, how many non-defective sets would you expect prior to the second defective?

$EX = \frac{r(1-p)}{p} = \frac{2 \times (1-0.05)}{0.05} = 38$