Math 2790: Assignment 1

Due on Tuesday, September 25^{th}

You may discuss these problems with others, but all solutions must be written individually. Please acknowledge any help you received. If you are completely stuck on a question, feel free to either see me or look at the hints on the course web page.

1. James and John stand on diagonally opposite corner squares of a 4 by 4 chessboard. For some reason, James is really angry with John, so he tries to catch him. They take turns moving, and on each turn, they move one square (either horizontally or vertically). James can catch John by moving onto the square that John is on. If James moves first, can he ever catch John? Explain.

2. At a party attended by 2001 people, various handshakes took place between the guests. Just for fun, everyone made a tally of the number of people that they shook hands with. Show that there must have been two people who shook the same number of hands.

3. At a popular restaurant in Dartmouth, n people sit around a circular table. The food is placed on a circular platform in the centre of the table, and this circular platform can rotate (most Chinese restaurants have this – the idea is that every person can take a little bit of each item without getting up from their seats or having to pass hot plates around the table). Each person orders a different entree, and it turns out that no one has the correct entree in front of her. Prove that it is possible to rotate the platform so that at least *two* people will have the correct entree.

4. Riham writes the numbers $1, 2, \ldots, 10$ on the board. She performs the following operation nine times: she picks two numbers at random from the board, erases them, and writes down $|a - b|$. Prove that an odd number will remain at the end.

5. Suppose that each square of a 3 by 7 chessboard is arbitrarily coloured indigo or teal. Prove that the board must contain a rectangle whose four corner squares are all the same colour. Does this result hold if the chessboard is 3 by 6? Explain your answer.

6. Eve and Oddie play a game on a 3 by 3 checkerboard, with black checkers and white checkers. The rules are as follows:

   i) They play alternately, with Eve moving first.

   ii) A turn consists of placing one checker on an unoccupied square of the board. A player may select either a white checker or a black checker.

   iii) When the board is full, Eve obtains one point for every row, column or diagonal that has an even number of black checkers, and Oddie obtains one point for every row, column or diagonal that has an odd number of black checkers.

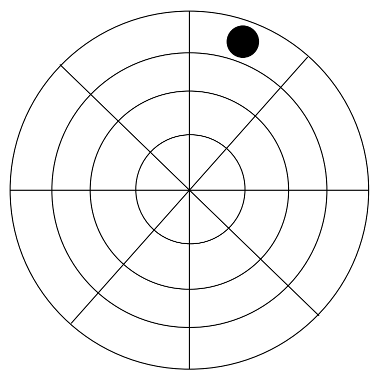
   iv) The player obtaining at least 5 of the 8 points wins.

   Describe a winning strategy for Eve (i.e., show that she can always win the game regardless of how well Oddie plays).

7. Alison and Ian play a game by alternately moving a disk on a circular board. The game starts with the disk already on the board as shown. A player may move either clockwise one position or one position toward the centre, but cannot move to a position that has been previously occupied. The last person who is able to move wins the game. Alison moves first.

   a) Find a winning strategy for Alison, and prove that it is indeed a winning strategy (i.e., no matter how well Ian plays, Alison will always be able to win).

   b) Is there a winning strategy for either of the players if the board is changed to five concentric circles with nine regions in each ring? If so, explain the strategy and justify your answer.



8. There is a rectangular pool table ABCD with side lengths m and n, where m and n are integers with $m < n$. I put the cue ball at corner A and shoot it at a 45 degree angle to the sides, and it reflects perfectly (at 45 degrees) off all the sides, as illustrated in the diagram, and never loses energy. The table has four pockets, one in each corner, and the ball will be sunk as soon as it hits a corner. The question is, given the integers m and n, how do you tell which pocket the ball hits first, and how many reflections will the ball make on the way? The example shown below is a 6 by 10 table and in this case, the ball makes 6 reflections before going into pocket C.

   If the table is 210 by 357, determine which pocket the ball will go into, and also determine how many reflections will take place. Prove the general case: what happens if the table is m by n?