Math 2790: Assignment 3

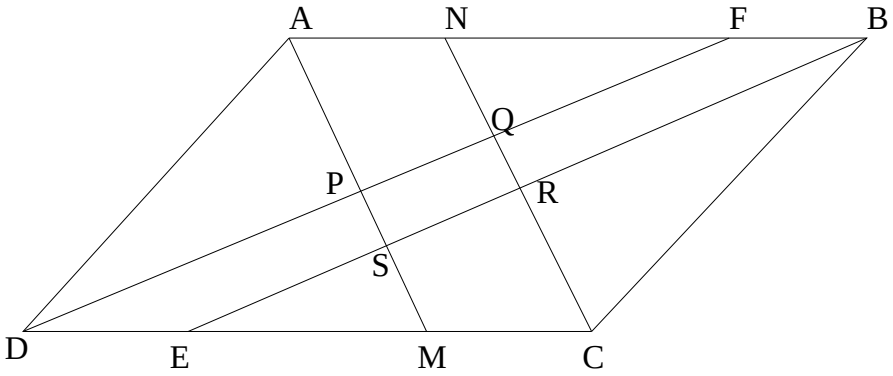
Due on Thursday, November 8th

(Formative Evaluation due on Tuesday, November 6th)

1. Let $\triangle ABC$ be an acute-angled triangle. Using Ceva's Theorem, prove that the three altitudes of $\triangle ABC$ are concurrent.

2. In a convex quadrilateral, each of its two diagonals divides the quadrilateral into two triangles of equal area. Prove that the quadrilateral is a parallelogram.

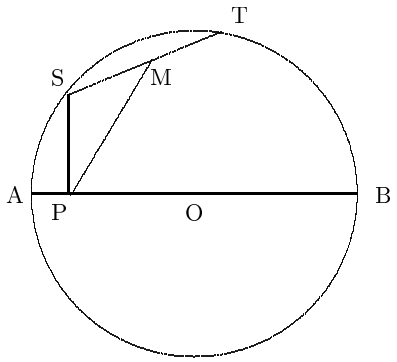
3. In parallelogram $ABCD$, $AB = a$ and $BC = b$, where $a > b$. The points of intersection of the angle bisectors are the vertices of quadrilateral $PQRS$.



   (a) Prove that $PQRS$ is a rectangle.

   (b) Prove that $PR = a - b$.

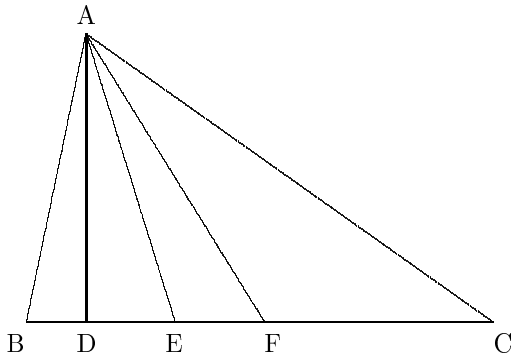
4. ST is any chord of length 2 on the semicircle with diameter 4. M is the midpoint of ST. P is the foot of the perpendicular from S to diameter AB. Determine $\angle SPM$.



5. Let I be the incentre of $\triangle ABC$. AI meets the circumcircle of the triangle at D. Prove that $DI = DB = DC$.

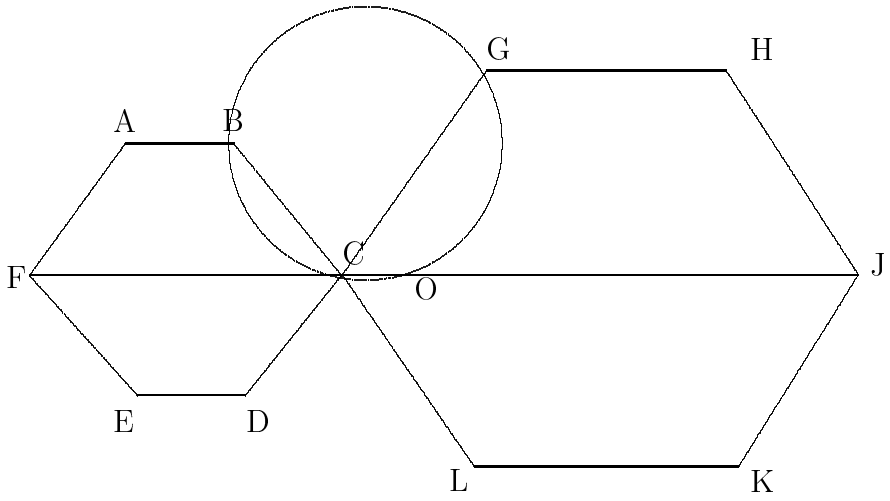
6. a) Let ABC be a right-angled triangle, with BC as its hypotenuse. Construct altitude AD, internal angle bisector AE, and median AF. Prove that AE bisects $\angle DAF$.

   b) Suppose that $AD = 28$ and $AE = 35$. Determine the area of $\triangle ABC$.



7. a) Equilateral triangle ABC is inscribed in a circle and a point P is on arc BC. Prove that $PA = PB + PC$.

   b) Two unequal regular hexagons $ABCDEF$ and $CGHJKL$ touch each other at C and are situated so that F, C and J are collinear. Let the circumcircle of $\triangle BCG$ meet FJ again at O. Show that $\triangle BOG$ is equilateral and that O is the midpoint of FJ.



8. Make up your own problem, in *any* topic of your choice. Provide your problem *and* a solution to the problem. Be creative!

BONUS: Captain Victoria finds her way to Treasure Island, which is circular in shape. She knows that there is treasure buried at the midpoint T of the segment joining the orthocentres of $\triangle ABC$ and $\triangle DEF$, where A, B, C, D, E, F are six palm trees on the shore of the island in some order unknown to Captain Victoria. What is the maximum number of points at which Captain Victoria has to dig in order to recover the treasure?