Tour 15 - the Euler Line

Recall the following:

The orthocentre H is the intersection point of the three altitudes of a triangle.

The centroid G is the intersection point of the three medians of a triangle.

The circumcentre O is the intersection point of the perpendicular bisectors of a triangle. Also, O is the centre of the circumcircle of the triangle.

Euler proved that H, G, and O are collinear, i.e., these three special points all lie on one common line. In addition, Euler proved that $HG = 2GO$, for all triangles. This is an incredible result!

Euler proved that triangles AHG and GOM are similar, and then he claimed that he was done.

Let's verify Euler's result. We need to prove the following four statements:

(a) $AG = 2GM$

(b) $AH = 2MO$.

(c) $\angle HAG = \angle OMG$

(d) Once we establish (a), (b), and (c), we're done! In other words, we have proven that H, G, and O must be collinear, with $HG = 2GO$.

Let's do a jigsaw once again. Break into groups of four. One group will do a), another group will do b), and the final group will do both c) and d).