Problems For Tour 2

1. Pick any 6 elements from the set $\{1, 2, \ldots 10\}$. Show that you can always find two elements that add up to 11.

2. Pick any seven points or on inside a regular hexagon of side length 1. Show that there are two points that are at most 1 unit apart.

3. A *lattice point* is a point whose coordinates are both integers. For example, $(19, 99)$ is a lattice point, but $(3, -2.5)$ is not. Pick any five distinct lattice points. Prove that it is possible to select two of these points such that its midpoint is also a lattice point.

4. Consider six points in the plane, of which no three are collinear (i.e., no line passes through three of these points). Draw lines that connect each point to every other point (there should be 15 lines in all), and colour each of these lines either turquoise or burgundy. Show that there must exist a *monochromatic* triangle (i.e., a triangle where all three sides have the same colour).

5. Suppose there are six people at a party. Show that among the six, there are either three mutual acquaintances or three mutual strangers.

6. Prove that some multiple of $\sqrt{2}$ lies within $\frac{1}{1000000}$ of an integer.