# Tour 21 - The Price is Right

**Problem: Determine the probability that Ryan wins the Dodge Dakota.**

Ryan can only move horizontally or vertically. The correct path involves five different squares, i.e., no square is repeated twice. So once Ryan determines that a square is part of the correct path, he will not come back to the same square on a subsequent move.

The probability of winning the game changes if we change the correct path to something else. So for this problem, assume that the path is up, right, right, up. (The correct path is indicated in the diagram by bold font).

| | | | | |
|---|---|---|---|---|
| 0 | 4 | 9 | 5 | **0** |
| 8 | 2 | **5** | **6** | **2** |
| 4 | 3 | **1** | 8 | 5 |
| 1 | 7 | 2 | 9 | 3 |
| 5 | 6 | 3 | 4 | 1 |

*Other Assumptions*

Assume Ryan is not an intelligent contestant. In other words, he moves completely randomly on each move. So on his first move, he has a one-fourth chance of moving to the 5. On his second move, he has a one-third chance to going to each of the three squares that he may choose from (since he can't go back to the middle square), and so on.

Assume Ryan is not a dumb contestant either. In other words, if he moves to an incorrect square (say down on his first move), then he won't step on the same incorrect square twice. So if Ryan makes a mistake on his first move, the probability that he moves to the 5 on his next move becomes one-third, and not one-fourth.

If Ryan makes a mistake, he is allowed to win another chance by playing a pricing game. He has to guess the correct price of an item from two possible choices. Assume that Ryan has a one-half chance of winning each pricing game.

There are three pricing games that Ryan has throughout the game. So if he misses a pricing game, he can try another one. However, once Ryan uses up his three pricing games, he can no longer make any mistakes on the board, or the game is over.

*Here are some questions to guide you along.*

Let $P_n$ be the probability of Ryan winning the game if he makes exactly $n$ mistakes. He is allowed to make up to three mistakes, so we are interested in $P_n$, for $n \leq 3$.

Determine $P_0$. Now see if you can figure out an elegant way to determine $P_1$ and $P_2$ without resorting to messy computation. (Hint: $P_1$ and $P_2$ are both integer multiples of $P_0$.)

Use this information to determine the probability that Ryan wins Pathfinder.