Problems For Tour 4

Here are some problems involving *Proof by Contradiction.*

1. Prove that $\sqrt{2}$ is irrational.

2. Let a, b, c, d, e, f be a sequence of six real numbers. Is it possible for this sequence to have the property that the sum of any two consecutive terms is positive and the sum of any three consecutive terms is negative? (2002 Euclid Contest).

3. A permutation $(a_1, a_2, \ldots, a_n)$ of the integers $1, 2, \ldots, n$ is said to be *fantastic* if $a_1 + a_2 + \ldots + a_k$ is divisible by k for *each* k from 1 to n. For example, $(3, 1, 2)$ is a fantastic permutation of $1, 2, 3$ because 3 is divisible by 1, $3 + 1$ is divisible by 2, and $3 + 1 + 2$ is divisible by 3. However, $(2, 1, 3)$ is not a fantastic permutation because $2 + 1$ is not divisible by 2.

   a) Show that no fantastic permutation exists for $n = 4$.

   b) Does a fantastic permutation exist for $n = 5$? Explain.

   c) What if we have $n = 2000$? How about $n = 2001$? Are there any fantastic permutations then? (2000 Euclid Contest)