Proof for Week 3

Due on Thursday, September 27^{th}

Find the flaw in the following argument, and clearly explain why this proof is not correct.

We will prove that **every person in this class has the same hair colour**, by showing that the following statement is true for all positive integers n: *for any group of n people in the class, every person in this group has the same hair colour.* If we can prove this statement is true for all positive integers n, then we will have proven that every person in this class has the same hair colour.

Clearly the result is true for $n = 1$. If the group only consists of one person, then everybody in that group has the same hair colour.

Suppose the result is true for $n = k$, i.e., that for *any* group of k people, every person in that group has the same hair colour. Now pick *any* group of $k + 1$ people from the class. Let's isolate one person from that group. Call him Patrick. So excluding Patrick, there are k people in the group, so by the induction hypothesis, these k people have the same hair colour. Say they all have blond hair. Now let's put Patrick back into the group and remove a different person. Call her Patricia. So excluding Patricia, there are k people in the group, so by the induction hypothesis, these k people also have the same hair colour. Since we know that $k - 1$ of these people have blond hair, it follows that everybody in this group must have blond hair. Therefore, all $k + 1$ of these people have blond hair.

Hence, we have proven that if the given statement is true for $n = k$, then it must also be true for $n = k + 1$. Since the statement is true for $n = 1$, by mathematical induction, it is true for all positive integers n. Therefore, every person in this class has the same hair colour.