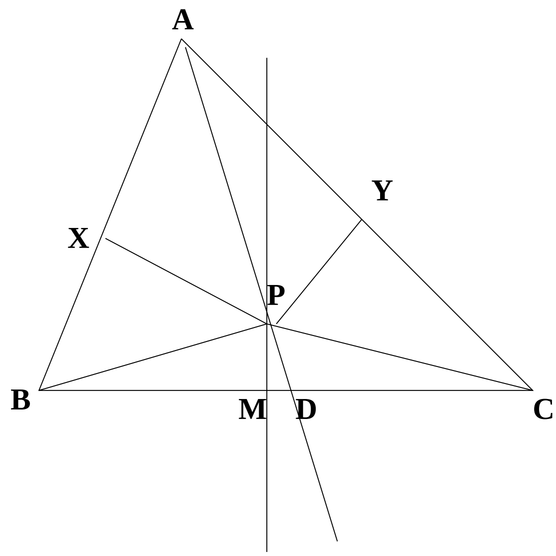
Proof for Week 8

Due on Thursday, November 1st

Here was our BOGUS proof that all triangles are isosceles.



Pick any arbitrary triangle ABC.

Let the internal angle bisector of $\angle A$ and the perpendicular bisector of side BC meet at point P, as shown. Construct PB and PC, and find points X and Y on AB and AC respectively so that $PX \perp AB$ and $PY \perp AC$.

Now, $\triangle APX \cong \triangle APY$, by Angle-Side-Angle (ASA) congruency test. Hence, $PX = PY$, and $AX = AY$. Since P is the perpendicular bisector of side BC, we have $BM = MC$, so by the Side-Angle-Side congruency test, $\triangle PMB \cong \triangle PMC$, and so $PB = PC$.

Finally, $\triangle PXB \cong \triangle PYC$ by the Hypotenuse-Side congruency test, since $PX = PY$, $PB = PC$, and $\angle PXB = \angle PYC = 90°$. Thus, $XB = YC$.

We have proven that $AX = AY$ and $XB = YC$. Therefore, $AX + XB = AY + YC$, which simplifies to $AB = AC$. Since we started with an arbitrary triangle ABC, and we proved that $AB = AC$, we conclude that for any triangle ABC, two of its sides must be equal. Therefore, we have proven that all triangle are isosceles, Q.E.D.

In class we proved that the flaw in this proof lies in the fact that P lies outside the triangle. We also briefly described how to prove this, using the Internal Angle Bisector Theorem. Your task for this weekly proof is to reproduce that argument, in your own words. Answer these two questions.

(a) If $\triangle ABC$ is isosceles (specifically, if $AB = AC$), what can you say about point P, the intersection of the angle bisector of $\angle A$ and the perpendicular bisector of side BC?

(b) Prove that if $\triangle ABC$ is not isosceles, then point P must lie outside the triangle.

Recall that the Internal Angle Bisector Theorem states that if AD is an internal angle bisector of $\angle BAC$, then $\dfrac{AB}{AC} = \dfrac{BD}{DC}$.