

Math 2400 - Numerical Analysis

Homework #1 Due September 25th

The purpose of the first assignment is to give you a chance to experiment with MATLAB and recall some calculus. Please show all your work for all questions. For the first two questions also provide printouts of the MATLAB commands leading to your solution. You may use the MATLAB `diary` command to create the printouts.

There are several sources of help in Matlab. There is the `help` command which you may use from the command line interface (just type `help command` to get for `command` or type just type `help` to get a list of topics) and there is a gui driven help system. You may also wish to take a look at the tutorials linked from the course homepage.

1. (a) Using the Matlab command line interface, create a 2×2 matrix named A with the following entries:

$$A = \begin{pmatrix} 2 & 4 \\ 7 & 9 \end{pmatrix}.$$

- (b) Type in the Matlab command `A.^2`.
- (c) Type in the Matlab command `A^2`.
- (d) What is the purpose of the dot.
2. (a) Using the Matlab command line interface, create a vector \vec{v} with the following structure. The first component of \vec{v} is $v_0 = 0$, the last component should be 3 and successive elements should be 0.1 apart. There are several ways to do this, you may want to look at the `linspace` command.
- (b) Create a vector \vec{w} with the property that the i^{th} component of \vec{w} is given by $v_i^3 + v_i + 1$.
- (c) Use \vec{v} and \vec{w} to plot the function, $y = x^3 + x + 1$ (look up the `plot` command in the help index) for $0 \leq x \leq 3$.
- (d) Try typing the command `z=sin(v)`.
- (e) Plot (very) approximately half of one period of the function $y = \sin(x)$.
3. Explain why the MATLAB command `sin(pi)` doesn't return 0. Explain the significance of the value returned by MATLAB (consider the effect on significant digits in future calculations).
4. Use 4 digit rounding with the usual formula¹ to find the roots of the following quadratic

$$x^2 + 62.1x + 1 = 0.$$

If the exact answers are taken to be

$$x_1 = -0.01610723 \quad x_2 = -62.08390$$

find the relative error of the approximation above using 4 digit rounding. Explain any unusual results.

5. Review Taylor's Theorem (page 21 of text): Construct the degree 3 Taylor polynomial $P_3(x)$ about $x_0 = 1$ approximating $\ln(x)$, the natural logarithm of x . Use this polynomial to approximate $\ln(1.1)$. Finally, using the truncation error (or remainder term) for this Taylor

¹ $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

polynomial, bound the error of your approximation to $\ln(1.1)$. How many digits of your approximation are correct.