Math 2400 - Numerical Analysis

Homework #2 Due October 16^{th}

This homework assignment will cover root finding and basic polynomial interpolation. Some computer programming will be required to complete this assignment. There are several links on the course home-page as well as the help facilities of Matlab to help with any difficulties. If you need assistance from me don't hesitate to ask.

Hand in printouts of all program listings as well as output with the homework assignment.

Keep in mind that many of the algorithms I am asking you to implement already exist in Matlab. You may use these preexisting functions to test you code, but you will get no marks for turning in an assignment that uses these functions.

1. Implement a bisection root finding method. Your program should accept two endpoints, a tolerance limit and a function for input. It should then output the final approximation and the number of iterations. Make sure that the program checks that the initial interval is acceptable for this method.
2. Implement a Newton's method root finding method. Your program should accept an initial guess, a tolerance(for the relative difference between successive approximations), a function and it's derivative for input. It should then output the final approximation and the number of iterations. Make sure that your program can't be stuck in an infinite loop or divide by 0.
3. Use the programs you have written to find roots for the following:

Function	Newton's method guess	Bisection Method Interval
$\tan(x) - 2x$	1.4	$a = 1$, $b = 1.4$
$65x^4 - 72x^3 - 1.5x^2 + 16.5x - 1$	1	$a = 0$, $b = 1$
$x^3 - 6x^2 + 12x - 8$	3	$a = 1$, $b = 3$

   Use 10^{-4} as your tolerance for both methods.
   Discuss the appropriateness of the methods and initial guess to the above problems.
4. In this question, we will find interpolating polynomials of degree at most 6 for the function $f(x) = e^{-2x}$ using equally spaced points and using the Chebyshev roots.
   (a) Construct (use the computer or do it by hand) a polynomial of degree at most five, $P_5(x)$, interpolating $f(x)$ at the points $x_k = 0.4k$, $k = 0, \ldots, 5$ (you should have equally spaced points between 0 and 2). Plot $P_5(x)$ with a solid line and plot the points $f(x_k)$ with a + for $0 < x < 2$.
   (b) Use a linear transformation to move the zeros of the degree 6 Chebyshev polynomial to the interval $[0, 2]$.
   (c) Construct a polynomial of degree at most five, $\bar{P}_5(x)$, interpolating $f(x)$ at the points found in 4b. Plot $\bar{P}_5(x)$ with a solid line and plot the points $f(x_k)$ with a + on the interval $0 < x < 2$.
   (d) Plot $f(x) - P_5(x)$ and $f(x) - \bar{P}_5(x)$ on the same graph for $0 < x < 2$. Explain why one of the polynomials has a lower maximum error.