## Math 2400 - Friday November 30

Homework #3 Due date to be discussed.

Hand in printouts of all program listings as well as output with the homework assignment.

1. Write a program for polynomial interpolation of function values. Your program should accept two vectors as input. A vector of x coordinates for interpolation and a vector of y values (the function values at the given x coordinates). Your program should then: Compute the divided differences table for an arbitrary set of data pairs.

Test your code by constructing a polynomial of degree at most 2 that interpolates  $f(x) = e^x$  at  $x_1 = 0$ ,  $x_2 = 1$ ,  $x_3 = 2$ . Provide the divided difference table, the polynomial and the value of  $P_2(0.5)$  in your solution.

You may use this code for the remainder of the assignment.

- 2. Write a program that inputs a vector of interpolation points,  $x_1, \ldots, x_n$  and the interpolation values  $y_1, \ldots, y_n$ . The program should then output the coefficients for the interpolating cubic spline with free boundary conditions. Use the code to find the interpolating cubic spline for the function  $y = \frac{1}{1+x^2}$  with  $x_1 = -2$ ,  $x_2 = -1$ ,  $x_3 = 1$  and  $x_4 = 2$ . Graph and compare the spline with the original function.
- 3. Use the function written for (1) to construct a cubic polynomial to approximate the function  $f = \frac{1}{1+x^2}$ . The constructed cubic should interpolate the function values at the points  $x_1 = -2$ ,  $x_2 = -1$ ,  $x_3 = 1$ ,  $x_4 = 2$ . Graph and Compare the approximation to that of the question (2). The approximating polynomials in this question and in question (2) are both cubics interpolating the same data. Why are the different?
- 4. Write a program which will accept two vectors as input, a vector of x values (not necessarily unique) and a vector of y values. The program will then plot the parametrically defined polynomial interpolating these points.

Test your code on the data set

i	1	2	3	4	5
$x_i$	-1	0	1	0	1
$y_i$	0	1	0.5	0	-1

5. Write a program which accepts four vectors as input. Two of the vectors will contain the values for the interpolation points  $x_i$ 's and  $y_i$ 's. The other two vectors will contain the control points  $\hat{x}_i$  and  $\hat{y}_i$ . The program will then draw the Bezier spline defined by the points. Test your code on the data

i	1	2	3	4	5
$x_i$	-1	0	1	0	1
$y_i$	0	1	0.5	0	-1
$\hat{x}_i$	-1	0	1	0	2
$\hat{y}_i$	1	1	0	-1	-1

6. Compare the results from question (5) and (4). What are any advantages one method could have over another.