Math 2400 - Numerical Analysis

Homework #4 Due November 20

In this assignment, we will examine nonlinear and linear least squares. You will need some data to test your program with, so go to the course web-page and download the data file hw4data. Once you have the file in your directory, you can type `load -mat hw4data` at the Matlab command prompt. This will load two variables t and y. We will assume that $y = ce^{kt}$ for some c and k. The data contains errors, so we use least-squares to find the best c and k.

1. Use linear least squares to find the best fit quadratic $y = c_1 + c_2x + c_3x^2$ for the given data.
2. Use non-linear least squares to find a c and k which minimizes $\sum_{i=0}^{m}(y_i - ce^{kt_i})^2$. You may use Gauss-Newton iterations in your non-linear solver. Plot the exponential cure and the data on the same graph. Mark the data points with an `x`.
3. In this section we will use linear least squares on transformed data.
   (a) Transform the data with the `log(x)` function to an appropriate form for linear least squares.
   (b) Use linear least squares to determine a straight line which best approximates the transformed data.
   (c) Use your result to find c and k
   (d) plot the resulting curve and the data points on the same graph. Mark the data points with an `x`.
4. Compare the accuracy of the three methods by calculating

$$\sum_{i=0}^{m}(y_i - ce^{kt_i})^2$$

for all cases and compare the results. Give a possible explanation for any differences found.

Hint: To see the effect of the transform to a linear system, consider the effect of the transform on the error of the i^{th} step.

$$y_i = ce^{kt_i} + \epsilon_i$$

We take the transform and we get

$$\ln(y_i) = \ln(ce^{kt_i} + \epsilon_i)$$

To see what is happening use a tayor expansion about $\epsilon = 0$