Math 2400 - Numerical Analysis

Homework #5 Due Dec 7

Hand in printouts of all program listings as well as output with the homework assignment.

1. Let $f(x)$ be a given function which can be evaluated at any point. In the following question, h refers to the step size or the distance between the equally spaced points used in the approximation.
   (a) Find a 2nd order method (i.e., truncation error $O(h^2)$) approximating $f'''(x_0)$. Give the formula as well as an expression for the truncation error.
   (b) Use your formula to find approximations to $f'''(0)$ for the function $f(x) = e^x$ employing values $h = 10^{-1}, 10^{-2}, \ldots, 10^{-9}$. Verify for larger values of h, your formula is indeed 2nd order accurate[1]. Which value of h gives the closest approximation to $e^0 = 1$.
   (c) For the formula derived in (a), how does the roundoff error behave as a function of h as $h \to 0$.
2. Consider the integral
$$\int_0^4 e^{-x^2}\, dx$$
   (a) Approximate the integral using the composite trapezoid method with $n = 2$.
   (b) Approximate the integral using Gaussian Quadrature and $n = 3$.
   (c) Given that the exact answer is 0.8862269120 find the relative error for both methods. Comment on the result.

[1] Apply the formula with one value of h and then $\frac{h}{2}$. If your formula is 2nd order, the error should be reduced by a factor of 4