1. Short Answer

2. We have the following information about a funcition $f(x)$.

   - $f(1) = 2$
   - The first order divided difference $f[x_1, x_2] = 2$ for all values $x_1 \neq x_2$.

   What can you say about this function.

3. Determine the missing entries in the following divided difference table.

$x_0 = 0.0$	$f[x_0] =$		
$x_1 = 0.4$	$f[x_1] =$	$f[x_0, x_1] =$	
$x_2 = 0.7$	$f[x_2] = 6$	$f[x_1, x_2] = 10$	$f[x_0, x_1, x_2] = \frac{50}{7}$

4. We wish to find an interpolating cubic polynomial for a function on the interval $[0, 3]$. If we wish to minimize the global error bound for $x \in [0, 3]$, what points should we choose for interpolation.

5. We are given the data set $\{(x_i, y_i)\}_{i=0}^n$ and wish to construct a **quadratic** spline to interpolate the data. Assume the spline is of the form,

$$S(x) = \begin{cases} S_0(x) & x_0 \leq x \leq x_1 \\ S_1(x) & x_1 < x \leq x_2 \\ \vdots & \vdots \\ S_{n-1}(x) & x_{n-1} < x \leq x_n \end{cases} \tag{1}$$

   where

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 . \tag{2}$$

   (a) Write down, but do not solve, equations which will make $S(x)$

      i. Interpolate the given data set.

      ii. Continuous.

      iii. Smooth (have a continuous derivative).

   (b) If all the above conditions are met, how many more conditions will be needed to completely specify $S(x)$ (There are a total of $3n$ unknowns in (1)).

6. Assume we have the data set $\{(x_i, y_i)\}_{i=0}^n$. We wish to determine the constants C_1 and C_2 which minimize the sum of the square of the error for the model $y = \frac{C_1}{x + C_2}$.

   (a) Give the nonlinear least squares equations which must be satisfied for optimal values C_1 and C_2. **Note:** Just give the equations which C_1 and C_2 must satisfy. You do not need to consider any iterative method for finding the values.

(b) Use $\frac{1}{y} = \frac{x}{C_1} + \frac{C_2}{C_1}$ to find a transform of the data which will allows the use of linear least squares.

(c) Find the normal equations need to solve for C_1 and C_2 which minimizes the error for the transformed data using linear least squares.

(d) If the error of the i^{th} data is has the form $y = \frac{C_1}{x+C_2} + \epsilon_i$, use an order one Taylor series about $\epsilon_i = 0$ to show how the transform modifies the error. What effect will this have on the resulting approximation.

7. Numerical differentiation Find an $O(h^2)$ approximation to $y'(x_0)$ using the values $y(x_0)$, $y(x_0 + h)$ and $y(x_0 + 2h)$. Find the error term as well.

8. Use three point Gaussian quadrature to approximate,

$$\int_0^1 e^{-x^2}\, dx$$

You may use the table below. You do not need to simplify your answer.

Points	Weighting Factors	Function Arguments
2	c1 = 1.000000000	x1 = -0.577350269
	c2 = 1.000000000	x2 = 0.577350269
3	c1 = 0.555555556	x1 = -0.774596669
	c2 = 0.888888889	x2 = 0.000000000
	c3 = 0.555555556	x3 = 0.774596669
4	c1 = 0.347854845	x1 = -0.861136312
	c2 = 0.652145155	x2 = -0.339981044
	c3 = 0.652145155	x3 = 0.339981044
	c4 = 0.347854845	x4 = 0.861136312

What is the highest order polynomial that this 3 point formula will provide an exact answer for?

9. Find c_1, c_2 and c_3 such that the rule

$$\int_0^1 f(x)\, dx = c_1 f(0) + c_2 f(0.5) + c_3 f(1)$$

is exact for all polynomials for degree 2 or less.

10. Develop a first order method for approximating $f"(x)$ that uses the data $f(x - h)$, $f(x)$ and $f(x + 3h)$ only. Find the error term.