Math 2400 – Practice Term Test

Note: In the real mid term test there will also be a short answer section with approximately 4 question. Each question will be a fill in the blank, true/false or multiple choice question.

1. The two roots of the quadratic $ax^2 + bx + c$ are given by,

$$x_\pm = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\,. \tag{1}$$

Here, x_+ corresponds to the root with the plus sign in front of the square root and x_- the other root.

   (a) Explain in one or two sentences what problems could result from using this equation when $4ac$ is very small compared to b^2.

   (b) These problems can be solved by calculating one of the roots using (1) and the other using,

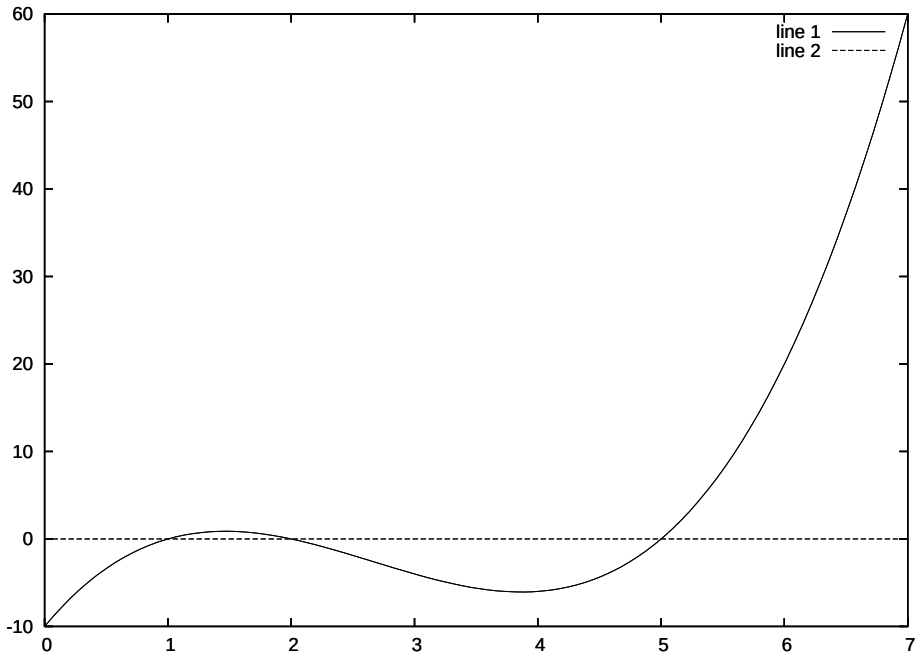
$$x_- x_+ = \frac{c}{a}\,. \tag{2}$$

   Which root should be calculated by equation (1) and why?

2. The graph of the function

$$f(x) = x^3 - 8\,x^2 + 17\,x - 10$$

is given by:



(a) If the points used for the bisection method are given by $a = 0$ and $b = 6$, what are the next two iterations of the bisection method?

(b) How many iterations are needed to ensure an error of less then 0.001?

(c) If an initial guess of $x_0 = 6$ is used, what is the first iteration of Newton's method?

3. To calculate any value of $\cos(x)$ we only need to be able to find $\cos(x)$ for $0 \leq x \leq \frac{\pi}{2}$ and use then use symmetry. We have the following exact values for the function $\cos(x)$ (and the decimal approximations) in this interval:

x	$\cos(x)$
0	1
$\frac{\pi}{6} \approx 0.52360$	$\frac{\sqrt{3}}{2} \approx 0.86603$
$\frac{\pi}{4} \approx 0.78540$	$\frac{1}{\sqrt{2}} \approx 0.70711$
$\frac{pi}{3} \approx 1.04720$	$\frac{1}{2}$
$\frac{\pi}{2} \approx 1.57080$	0

(a) Use divided differences to find the unique polynomial of degree at most 4 which interpolates these points.

(b) Find a bound on the maximum error of this approximation. Note for this bound, you may use $|x - x_i| \leq \frac{\pi}{2}$ even though this gives a very rough estimate.

(c) What interpolation points would we use to minimize the global error?

4. Circle the apprpriate descriptions below: The function defined by:

(a)
$$S(x) = \begin{cases} 10 - 5x + 2x^2 + x^3 & 0 \leq x < 1 \\ 8 + 2(x-1) + 5(x-1)^2 + 10(x-1)^3 & 1 \leq x \leq 2 \end{cases}$$

Free Cubic Spline Clamped Cubic Spline Not a cubic spline

(b)
$$S(x) = \begin{cases} 10 - 5x + x^3 & 0 \leq x < 1 \\ 6 - 2(x-1) + 3(x-1)^2 - (x-1)^3 & 1 \leq x \leq 2 \end{cases}$$

Free Cubic Spline Clamped Cubic Spline Not a cubic spline

(c)
$$S(x) = \begin{cases} 10 - 5x + x^3 & 0 \leq x < 1 \\ 7 + 2(x-1) + 3(x-1)^2 - (x-1)^3 & 1 \leq x \leq 2 \end{cases}$$

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5. Construct a parametric interpolating polynomial which passes through the following points:

i	1	2	3	4
x_i	-1	0	1	0
y_i	0	1	0.5	0