List of Potential MATH 1002 Projects

General instructions: you are required to create a poster to assist in your presentation, so that students can come back to it and have another look at some of the details. You need to give a 20 minute classroom presentation and write a short report.

(1) **Mathematics and Decision Making** The part of mathematics called "Game Theory" helps us to analyze decision making when two or more people are involved and their choices influence the outcome of the process for everybody. A famous problem of this type is the "prisoners' dilemma". Two people have been arrested related to the same crime. If only one of them confesses, that person will end up with 20 years in jail; if they both confess, they will each only get 2 years jail time; if neither one of them confesses, they will both serve 6 years. If you cannot communicate with your partner, what should you do? Suppose that you and your partner are regularly involved in organzied crime, and this situation happens more frequently; can you possibly learn form your partner's behaviour?

For this project, you will learn how to turn a situation like this into a table with pairs of numbers indicating how much each party likes a given situation, and you will learn the reasoning strategies to come to a good conclusion. It would be nice if you could apply this to a situation that you have invented (or even better, a real life situation from the news).

Resource: Chapter 5 in "Game Theory, A Nontechnical Introduction" by Morton D. Davis.

(2) **Rigid Symmetry and Tilings of the Plane** Suppose that you want to make a pattern that keeps repeating itself, such as when you tile a floor or make wall paper. What shape can the tiles have? What kind of symmetry patterns can it have? In this project you will look at rotation and reflection symmetry and see which combinations can exist.

What could you do if you just wanted to make a freeze pattern?

You will have a chance to make your own patterns with different kinds of symmetries. But you will also look at art work from various cultures and be able to recognize the different types of symmetries.

(3) **Fine Symmetry of Scale and Fractals** Fractals are a fairly recent mathematical invention. The reason for this is that we really need computers to draw good pictures. Mathematicians in the past did know about them, but they could not study them in as much detail as we can do now.

Euclidean geometry always has to simplify natural objects before you can discuss them. For example, it may represent a horse as a ball. Fractals are good to model natural objects, because they have the same level of complexity no matter how much you zoom in. So you can model things that are more

complicated. The world of the third episode of Star Wars was completely made up using fractals!

The nice thing is that you only need a couple of equations to describe a fractal! There are several ways to construct fractals, and in this project you will learn about the most basic one. You will be able to use part of Chapter 6 in our book, as well as the book "Fractals Everywhere". There is a applet on the internet called Fractalina that you can use to make your own fractals. The question I want you to ask yourself is: how can you make a fractal that looks very close to an object that you would like to model?

(4) **Counting and the Abacus in various cultures** Many societies have used mechanical devices to be able to add and multiply numbers. Both western societies and Asian societies have used various versions of the abacus. The Inca used a quipu, consisting of string with various types of knots for the same purpose.

Some of the mathematics you will need and learn: Multiplication algorithms, counting in bases different from 10. Questions you should answer: How was the abacus used and by whom? (Explain how you could use it to multiply, add, etc.) Are they still in use today? By whom? What is its role in society?

Resources: there are several good internet sites

(5) **Women in Mathematics** Who were the first women involved in mathematics, that we know about? How was their work received? What kind of role did they play in the mathematical society? What were the obstacles they faced? Can you describe some of the mathematical work they have done? What are some issues women in mathematics face today? What motivates them?

Resources: there are several very good websites such as

www.agnesscott.edu/lriddle/women/women.htm

There are also many books in the library with index QA 27.5.

(6) **Penrose Tilings** Physicists studying various metalic structures have found structures that have 5-fold and 8-fold symmetry. This is not possible for a regular crystal structure. (Try creating a sidewalk pavement with pentagons or octagons, you will see that you always end up with gaps.) This is not possible with crystals (where all units have the same shape), but it is possible with quasi crystals, where we can have two shapes. An example of a pair of shapes that will let you create a pavement or tiling with five-fold symmetry are Penrose's kites and darts.

For tilings without rigid symmetry. Describe the basic tiles and how they fit together. Describe the algorithms to construct Penrose tilings. What are their special properties? Tell us more about Sir Roger Penrose. Create some of your own Penrose Tilings. Consider other tiles for aperiodic tilings.

The Penrose tiles were made out of golden rectangles and gnomons. Can you use them in different ways to make interesting tilings?

Resources: There are various websites with information on this topic as well as several applets to create Penrose tilings.

(7) **The Four Colour Theorem** Explain the problem. Give some examples. How many colours do you need for a map of Canada? How many do you need for the continent fo Africa? Describe some aspects of the proof. What is special about this proof? How is this changing the way we do mathematics? Are you convinced by this type of proof? Suppose that you had to give the map for a world on the surface of a donut (the technical mathematical term is "torus"). How many colours would you need? Can you think of other surfaces and how many colours they would need?

(8) **Graph Theory and Scheduling** When you need to organize activities such as mail delivery, garbage collection, or street cleaning, you need to come up with a systematic way to visit all streets in your city, hopefully without having to traverse any of them more than once. Is this always possible? How can you do this?

On the other, if you need to visit a number of important points of interest, such as the grocery store, the post office, the coffee shop, and the cinema, you are interested in creating a route that lets you visit all of these with a minimal amount of travelling distance.

Both of these types of problems have been studied by graph theorists. Graph theory is a branch of mathematics, and for this project you will learn Euler Circuits, Eulerization, Fleury's Algorithm, and Hamiltonian Paths.

As you learn about these important techniques, you should also find out some historical information about Euler, Fleury, and Hamilton.

(9) **The development of mathematics in Japan** Give an overview of the development of mathematics in Japan. Discuss how the development of mathematics is related to the development of the Japanese culture in general. What kind of results were discovered first? Why were they of interest to the Japanese people?

Who were the important people in the development of Japanese mathematics? When was there communication with Western mathematicians?

Look for some interesting results or developments and report on them.

Resources: There are books in the QA 27 section of the library. One of them is "A History of Japanese Mathematics" by David Eugene Smith and Yoshio Mikami. There is also information available on the internet.

(10) **The development of mathematics in China** Give an overview of the development of mathematics in China. Discuss how the development of mathematics is related to the development of the Chinese culture in general. What kind of results were discovered first? Why were they of interest to the Chinese people?

Was there mathematically inspired art? Show some examples and if possible discuss the mathematics briefly.

Who were the important people in the development of Chinese mathematics? When was there communication with Western mathematicians?

Look for some interesting results or developments and report on them.

Resources: There are books in the QA 27 section of the library. One of them is "Chinese Mathematics A Concise History" by Li Yan and Du Shiran. There is also information available on the internet.

(11) **The development of mathematics in India** Give an overview of the development of mathematics in India. Discuss how the development of mathematics is related to the development of the Indian culture in general. What kind of results were discovered first? Why were they of interest to the Indian people?

Was there mathematically inspired art? Show some examples and if possible discuss the mathematics briefly.

Who were the important people in the development of Indian mathematics? When was there communication with Western mathematicians?

Look for some interesting results or developments and report on them.

Resources: There are books in the QA 27 section of the library. One of them is "Geometry in Ancient and Medieval India" by T.A. Sarasvati Amma (but there are more on other mathematical subjects). There is also information available on the internet.

(12) **Babylonian Mathematics** Give an overview of the development of mathematics in the Middle Eastern countries and the Arabic peninsula. Discuss how the development of mathematics is related to the development of the Babylonian culture in general. What kind of results were discovered first? Why were they of interest to the Babylonian people?

Was there mathematically inspired art? Show some examples and if possible discuss the mathematics briefly.

Who were the important people in the development of Babylonian mathematics? When was there communication with Western mathematicians?

Look for some interesting results or developments and report on them.

Resources: There are books in the QA 27 section of the library. There is also information available on the internet.

(13) **The mathematics of line drawings** Various cultures in Africa and India include line drawings as part of their story telling or decorating techniques. There are various interesting mathematical aspects of these drawings.

Many of them can be drawn with just one line that begins and ends at the same point. How could you quickly see whether a particular drawing can possibly be drawn in that way? (This is a graph theory question.)

Another aspect is that many of these figures have further symmetry. Look for that symmetry and try to describe it.

Finally, the Indian drawings are often make according to a particular "recipe". There are several to create larger or more intricate drawings from smaller ones. Learn more about the mathematics behind these systems.

You will see in this project that people can use some pretty complicated mathematical concepts without ever calling it "mathematics". Does this say something about the nature of mathematics and how it is part of us?