**4.** Let $\mathbf{F}([-1,1]) = \{f \mid f : [-1,1] \to \mathbf{R}\}$ be the vector space of real-valued functions defined on $[-1,1]$. Recall that the zero of $\mathbf{F}[-1,1]$ is the function that has the value 0 for all $x \in [-1,1]$.

Define three functions in $\mathbf{F}([-1,1])$ by $f(x) = 1+x$, $g(x) = x+x^2$ et $h(x) = x+x^2+x^3$ and let $W = \operatorname{span}\{f, g, h\}$.

a) Show that $f$, $g$ and $h$ are linearly independent.

b) Find a basis for $W$ and the dimension of $W$.

c) If $j(x) = 1 - x^2 + x^3$ show that $j \in W$.

d) What is $\dim \operatorname{span}\{f, g, h, j\}$?

**5.** Supoose $v_1 = (1, 2, 1)$ and $v_2 = (-1, 1, -1)$ and let $W = \text{span}\{v_1, v_2\}$.

a) Show that $\{v_1, v_2\}$ is an orthogonal set. Is $\{v_1, v_2\}$ linearly independent? (Give reasons)

b) Give a complete geometric description of $W$.

c) Find an orthonormal basis $B$ of $W$, and hence find $\dim W$.

d) Extend the basis $B$ to an orthogonal basis of $\mathbf{R}^3$.

**6.** Suppose $\{u, v\}$ is a linearly independent set in vector space $V$, and suppose $w \in V$ is such that $\{u, v, w\}$ is linearly **de**pendent.

a) State carefully what "$\{u, v\}$ is linearly independent" means. (i.e. give the definition.)

In the next two parts, either show that the statement is always true, or give a counterexample (in $\mathbf{R}^2$ or $\mathbf{R}^3$) to show it isn't always true.

b) $w \in \operatorname{span}\{u, v\}$.

c) $u \in \operatorname{span}\{v, w\}$.