3. Consider the vector space

$$\mathbf{F}[0, 2\pi] = \{f \mid f \text{ is a real-valued function defined on } [0, 2\pi]\}$$

Recall that the zero of $\mathbf{F}[0, 2\pi]$ is the function that has the value 0 for all $x \in [0, 2\pi]$.

a) Show that $\{1, \cos 2x, \sin 2x\}$ is linearly independent in $\mathbf{F}[0, 2\pi]$.

b) If $z = \cos x + i \sin x$, use a well-known trigonometric identity to show that $|z| = 1$.

c) Use (b) and De Moivre's theorem, i.e., $(\cos x + i \sin x)^n = \cos nx + i \sin nx$, to show that $\cos 2x = 2\cos^2 x - 1$

d) Deduce from (c) that $\cos^2 x \in \operatorname{span}\{1, \cos 2x, \sin 2x\}$.

e) Is $\{\cos^2 x, 1, \cos 2x, \sin 2x\}$ linearly independent in $\mathbf{F}[0, 2\pi]$? Explain.

4. Let $W = \{(x, y, z) \in \mathbf{R}^3 \mid x + z = 0\}$

a) Give a complete geometric description of W.

b) Is W a subspace of $\mathbf{R}^3$? Give reasons for your answer.

c) Find a basis for W (you must check at least one of the two conditions to be satisfied) and hence find the dimension of W.

d) Is your chosen basis orthogonal? Justify your answer.

5. In each case, give an explicit example of:

a) a linear system of 2 equations in 2 unknowns with no solutions. Sketch the graphs of the equations of your system.

b) a linear system of 2 equations in 3 unknowns with at least 2 different solutions. (You must explicity give two different solutions.)

c) any linear system with a unique solution. Give the unique solution.