UNIVERSITÉ D'OTTAWA • UNIVERSITY OF OTTAWA

Sciences • Science

Mathématiques et statistique
Mathematics and Statistics

MAT 1341B Final Exam

April 26, 2002 Duration: 3 hours.
Instructor: Barry Jessup.

Family Name:______________________

First Name:________________________

Student number:____________________

1	
2	
3	
4	
5	
6	
7	
8	
9	
sub-total	
10	
11	
12	
13	
14	
Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 3 hours to complete this exam.
2. This is a closed book exam, and no notes of any kind are allowed. **The use of calculators**, cell phones, pagers or any text storage or communication device **is not permitted.**
3. Read each question carefully -you will save yourself time and unnecessary grief later on.
4. Questions 1 to 9 are multiple choice. These questions are worth 2 points each and no part marks will be given. Please record your answers in the space provided above.
5. Questions 10 – 14 require a complete solution, and are worth 6 points each, so spend your time accordingly.
6. **The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct.**
7. **You must answer these questions in the space provided.** Use the backs of pages if necessary.
8. Where it is possible to check your work, do so.
9. Bonne chance! Good luck!

1. The eigenvalues of the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & -1 \\ 0 & 2 & -3 \end{bmatrix}$ are:

A. 2, 3, 4
B. -3, 3, 4
C. 0, 1, 3
D. -3, 0, 4
E. -1, -2, 1
F. 0, 0, 2

2. Find the main diagonal of the inverse of $\begin{bmatrix} 1 & -2 & -3 \\ -2 & 2 & 4 \\ -3 & 0 & 2 \end{bmatrix}$.

A. $(2,\ -7/2,\ -1)$
B. $(5/2,\ 7/2,\ 3/2)$
C. $(2,\ 1,\ -1)$
D. $(-1,\ -7/2,\ 3)$
E. $(7/2,\ 2,\ -1)$
F. $(2,\ 1,\ -7/2)$

3. Solve the following equation for A:

$$A^t - [1\ 0\ 0]^t[0\ 1] = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$$

A. $\begin{bmatrix} 0 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 2 & 3 \\ 4 & 4 & 6 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 6 \end{bmatrix}$ E. $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ F. $\begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$

4. For a non-homogeneous system of 12 equations in 15 unknowns, answer the following three questions:

- ∘ Can the system be inconsistent?
- ∘ Can the system have infinitely many solutions?
- ∘ Can the system have a unique solution?

A. No, Yes, No.
B. Yes, Yes, Yes.
C. Yes, Yes, No.
D. No, No, No.
E. Yes, No, Yes.
F. No, No, Yes.

5. Let $U = \text{span}\{(1,-2,3,4),(-3,6,-5,-16),(-1,2,-5,-2)\}$ be a subspace of $\mathbf{R}^4$. Then $\dim U^{\perp}$ is

A. 0
B. 1
C. 2
D. 3
E. 4
F. 5

6. The dimension of $S = \{A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{22} \mid A = A^t\}$ is:

A. 0
B. 1
C. 2
D. 3
E. 4
F. 5

7. Which of the following statements are true?

(1) Each spanning set for $\mathbf{R}^n$ has exactly n vectors.

(2) If $\{u, v, w\}$ is linearly independent, then $\{u, v\}$ is also linearly independent.

(3) If A is an $n \times n$ matrix, then $\det A = (-1)^n \det(A^t)$.

(4) If A is an $n \times n$ matrix, then $\dim \mathrm{col}\ A = n$.

(5) If A is an $n \times n$ matrix, the $\dim \ker A = n - \operatorname{rank} A$.

(6) The set of $n \times n$ diagonal matrices is a subspace of the vector space of of all $n \times n$ matrices.

A. All six are true.
B. (2), (5) and (6).
C. (1), (2) and (4).
D. (3), (2) and (6).
E. (4), (5) and (6).
F. (2), (3) and (5).

8. Let A and B be two matrices such that $\det A = -2$ and $\det(B^t) = 3$. Find $\det(AB)$.

A. 6
B. 1
C. 5
D. -5
E. -6
F. -3/2

9. What is the dimension of the subspace of $\mathbf{R}^3$ spanned by $(1,\ 1,\ 1)$, $(-1,\ 1,\ -1)$, $(1,\ 1,\ 3)$ and $(0,\ 2,\ 1)$?

A. 0
B. 1
C. 2
D. 3
E. 4
F. These vectors do not span a subspace.

10. Consider the linear system

$$\begin{array}{rcrcrcl} x & - & y & - & z & = & 1 \\ x & & & + & z & = & p \\ & & y & + & qz & = & 3 \end{array}$$

a) If $[A\,|\,b]$ is the augmented matrix of the system above, find $\operatorname{rank} A$ and $\operatorname{rank}[A\,|\,b]$ for all values of p and q.

10b) Find all p and q so that this system has

i) a unique solution,

ii) infinitely many solutions, and

iii) no solutions.

10c) In case (ii) above, give a complete geometric description of the set of solutions.

11. Let $A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$.

a) Find $\det(A - 2I_3)$ and hence conclude that 2 is an eigenvalue of A.

b) Find a basis of $E_2 = \{x \in \mathbf{R}^3 \mid Ax = 2x\}$.

11 c) Check that $(1,1,1)$ is an eigenvector of $A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$.

11 d) If possible, find an invertible matrix P such that $P^{-1}AP = D$ is diagonal and give this diagonal matrix D.

12. Let $W = \{(x, y, z) \in \mathbf{R}^3 \mid x - y - 2z = 0\}$.

a) Find a basis of W and give the dimension of W.

b) Find an orthogonal basis of W.

12c) Find the best approximation to $(1, 0, 0)$ by vectors in W.

13. a) Let A be a real $n \times n$ matrix. Give 4 additional different statements which are equivalent to:

$$Ax = b \text{ is consistent for all } b \in \mathbf{R}^n.$$

(I)

(II)

(III)

(IV)

13b) State (in the box) whether the following are true or false. If true, explain why, if false, give a numerical example to illustrate.

i) If a 5 by 3 matrix A has rank 3, the system $Ax = 0$ has a unique solution. $\boxed{}$

ii) If a 5 by 5 matrix A satisfies $A^3 = 0$, but $A \neq 0$, then A is invertible. $\boxed{}$

14. Let $u = (1, 0, 0, 1), v = (0, 1, 1, 0)$ and $w = (1, 0, 0, -1)$ be vectors in $\mathbf{R}^4$.

a) Carefully show, by any method, that $\{u, v, w\}$ is linearly independent.

b) Find a vector $z \in \mathbf{R}^4$ so that $\{u, v, w, z\}$ spans $\mathbf{R}^4$, and support your answer.

14c) If $(x_1, x_2, x_3, x_4) \in \text{span}\{u, v, w\}$, find formulae in terms of x_1, x_2, x_3 and x_4 for the scalars c_1, c_2 and c_3 such that $(x_1, x_2, x_3, x_4) = c_1u + c_2v + c_3w$.

(blank)