MAT 1341, INTRODUCTION TO LINEAR ALGEBRA, WINTER 2003

Minitest 1, February 7

Prof. P. Selinger

FAMILY NAME: ________________ **FIRST NAME:** ________________ **ID:** ________________

Record your answers to questions 1-4 here:

Question:	1	2	3	4
Answer:				

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this test.
2. This is a closed book test, and no notes of any kind are allowed. The use of calculators is neither required nor permitted.
3. The test has 8 questions.

   Questions 1–4 are multiple choice questions, each worth 2 points. Record the **letters** corresponding to your answers of questions 1–4 in the boxes above. Do not use pencil. For the multiple choice questions, only this page will be graded and no part marks will be given.

   Questions 5–6 require a short answer and count 3 points each.

   Questions 7–8 require a detailed answer and count 3 points each. Please write legibly and argue carefully. You can write on the back of the page if necessary.

Question 1. For which values of a and b does the system

$$\begin{cases} -x + 3y + 2z = -8 \\ x + z = 2 \\ 2x + 2y + az = b \end{cases}$$

have more than one solution?

A. if $a = -4$ and $b \neq 0$.

B. if $a \neq -4$ and $b \neq 0$.

C. if $a = 4$ and $b = 0$.

D. if $a \neq 4$ and $b \neq 0$.

E. if $a = 4$ and $b \neq 0$.

F. if $a = -4$ and $b = 0$.

Question 2. Let $A = \begin{pmatrix} 3 & 1 & 0 \\ 2 & 3 & -1 \\ 0 & 2 & -1 \end{pmatrix}$. Then the main diagonal of A^{-1} is:

A. $1, 3, -7$.

B. $-1, -3, -6$.

C. $1, -3, -7$.

D. $-1, 3, -6$.

E. $-1, -3, -7$.

F. $1, 3, -6$.

Question 3. For which values of a does the matrix $\begin{pmatrix} 1 & -a & 2 \\ 0 & 1 & -2 \\ 2 & 1 & a \end{pmatrix}$ have rank 2?

A. $a = -3/2$ and $a = 1$.

B. $a = 2/5$.

C. No value of a.

D. $a = 3/4$ and $a = -1/2$.

E. $a = -4/3$.

F. $a = 3/4$.

Question 4. Given a non-homogeneous system of 5 equations in 7 unknowns, answer by yes or no the following three questions and indicate which combination of answers is right.

• Can the system have no solution?

• Can the system have infinitely many solutions?

• Can the system have a unique solution?

A. No, No, No.

B. Yes, Yes, Yes.

C. No, No, Yes.

D. Yes, Yes, No.

E. No, Yes, Yes.

F. Yes, No, Yes.

Questions 5 and 6 are short answer questions.

Question 5 (3 points). Find scalars $a, b, c \in \mathbb{R}$ such that $au_1 + bu_2 + cu_3 = w$, where

$$u_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad w = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}.$$

Answer: $a =$ $\boxed{}$ $b =$ $\boxed{}$ $c =$ $\boxed{}$

Question 6 (3 points). Row reduce the following matrix to row canonical form:

$$\begin{bmatrix} 1 & 1 & 1 & -3 & 2 \\ 1 & 2 & 0 & -4 & -1 \\ 2 & 1 & 3 & -5 & 0 \end{bmatrix}.$$

Answer:

Questions 7 and 8 require a detailed answer. Show all your work. You can use the backs of pages if necessary.

Question 7 (3 points). Find a matrix A such that $AB = C$, where

$$B = \begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 8 & 4 \\ -3 & -1 \end{pmatrix}.$$

Question 8 (3 points). Consider the homogeneous system $Ax = 0$, where

$$A = \begin{pmatrix} -1 & 2 & 2 & 3 \\ 2 & -1 & 2 & 0 \\ 2 & 0 & 4 & 2 \end{pmatrix}.$$

Find a basis for the solution space of this system.