MATH/CSCI 2112, DISCRETE STRUCTURES I, FALL 2005
Handout 4: Problems for Predicate Logic

Problem 1. Using the following predicates, translate the sentences below into predicate logic.

$A(x)$	-	x is an artist
$E(x)$	-	x is an engineer
$B(x)$	-	x is a book
$U(x, y)$	-	x understands y
$W(x, y)$	-	x can write y

(a) No engineer can write a book unless they understand it.

(b) Engineers can only write books that they do not understand.

(c) Every artist can understand some book that Tom can write.

(d) Tom can write a book only if every artist understands it.

Problem 2. Using the following predicates, translate the sentences below into English.

$L(x)$	–	x is a lion.
$T(x)$	–	x is a tiger.
$A(x)$	–	x is an animal.
$E(x, y)$	–	x eats y.
$H(x, y)$	–	x hunts y.

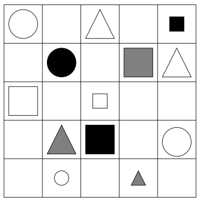
(a) $\exists x(L(x) \wedge \forall y(A(y) \longrightarrow H(x, y) \wedge \exists z(T(z) \wedge E(z, y))))$

(b) $\exists x(L(x) \wedge \exists y((\sim A(y) \vee H(x, y)) \wedge \sim \exists z(T(z) \wedge E(z, y))))$

(c) $\exists x(L(x) \wedge \forall y((A(y) \wedge H(x, y)) \longrightarrow E(x, y)))$

(d) $\exists y(A(y) \wedge \forall x((L(x) \wedge E(x, y)) \longrightarrow H(x, y)))$

Problem 3. Translate each of the below sentences into predicate logic, using the predicates $\texttt{Triangle}(x)$, $\texttt{Circle}(x)$, $\texttt{Square}(x)$, $\texttt{White}(x)$, $\texttt{Grey}(x)$, $\texttt{Black}(x)$, $\texttt{Large}(x)$, $\texttt{Small}(x)$, $\texttt{RightOf}(x, y)$, $\texttt{Above}(x, y)$, $\texttt{SameColorAs}(x, y)$. Also decide whether each sentence is true or false, referring to the instance of Tarski's world shown in the picture.



(a) There are no black triangles.

(b) All white triangles are large.

(c) All large triangles are white.

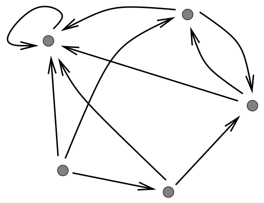
(d) There is a white triangle next to a grey square.

(e) Every large circle is the same color as some triangle.

(f) All squares that are below some circle are next to a white square.

Problem 4. A *directed graph* consists of vertices (dots) connected by arrows. A graph defines an interpretation, where the domain is the set of vertices, and we write $A(x, y)$ if there is an arrow from x to y.

(a) Referring to the following graph, decide which of the below sentences are true and which ones are false.



(A) $\forall x. \exists y. A(x, y)$.

(B) $\forall y. \exists x. A(x, y)$.

(C) $\exists x. \forall y. A(x, y)$.

(D) $\exists y. \forall x. A(x, y)$.

(E) $\forall x. \forall y. (A(x, y) \longrightarrow A(y, x))$.

(F) $\exists x. \exists y. (A(x, y) \wedge A(y, x))$.

(G) $\exists x. (A(x, x))$.

(b) Find an example of a graph that makes sentence (A) false and (B) and (C) true.

(c) Find an example of a graph that makes (A) and (C) false and (B) true.

Problem 5. Identify the free and bound variables in each of the following formulas. Also standardize the variables apart.

(a) $\forall x. (\exists z. (A(x, y, z) \wedge (\exists x. B(x, y, z)) \wedge C(x, z)))$.

(b) $\forall p. \forall q ((\exists p. A(p, q, r)) \implies (\exists q. A(p, q, r)))$.

Problem 6. Which of the following statements are true in the domain of the natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$?

(a) $\forall x. \exists y. (2 + y \leqslant x)$.

(b) $\exists x. \forall y. \exists z. (xz = y)$.

(c) $\exists x. \exists y. \exists z. (x > 1 \wedge y > 1 \wedge z > 1 \wedge x^2 + y^2 = z^2)$.

(d) $\exists x. \exists y. (2x = 2y + 1)$.