MATH/CSCI 2112, DISCRETE STRUCTURES I, FALL 2005
Handout 6: Rules for Program Correctness Proofs

Precondition strengthening

$$\frac{P_1 \Rightarrow P \qquad \{P\}\,C\,\{Q\}}{\{P_1\}\,C\,\{Q\}}$$

Postcondition weakening

$$\frac{\{P\}\,C\,\{Q\} \qquad Q \Rightarrow Q_1}{\{P\}\,C\,\{Q_1\}}$$

Conjunction rule

$$\frac{\{P_1\}\,C\,\{Q_1\} \qquad \{P_2\}\,C\,\{Q_2\}}{\{P_1 \wedge P_2\}\,C\,\{Q_1 \wedge Q_2\}}$$

Disjunction rule

$$\frac{\{P_1\}\,C\,\{Q_1\} \qquad \{P_2\}\,C\,\{Q_2\}}{\{P_1 \vee P_2\}\,C\,\{Q_1 \vee Q_2\}}$$

Sequencing rule (concatenation)

$$\frac{\{P\}\,C_1\,\{Q\} \qquad \{Q\}\,C_2\,\{R\}}{\{P\}\;C_1; C_2\,\{R\}}$$

Modified concatenation rule

$$\frac{\{P\}\,C_1\,\{Q\} \qquad Q \Rightarrow R \qquad \{R\}\,C_2\,\{S\}}{\{P\}\;C_1; C_2\,\{S\}}$$

Assignment rule for variables

$$\frac{}{\{Q[E/x]\}\; x := E\,\{Q\}}$$

Modified assignment rule for variables

$$\frac{}{\{E \text{ well-defined}, Q[E/x]\}\; x := E\,\{Q\}}$$

If-then-else rule, forward

$$\frac{\{P \wedge B\}\,C_1\,\{Q\} \qquad \{P \wedge {\sim}B\}\,C_2\,\{R\}}{\{P\}\;\textbf{if } B \textbf{ then } C_1 \textbf{ else } C_2\,\{Q \vee R\}}$$

If-then-else rule, backward

$$\frac{\{P\}\,C_1\,\{Q\} \qquad \{R\}\,C_2\,\{Q\}}{\{(P \wedge B) \vee (R \wedge {\sim}B)\}\;\textbf{if } B \textbf{ then } C_1 \textbf{ else } C_2\,\{Q\}}$$

If-then-else rule, both ways

$$\frac{\{P \wedge B\}\,C_1\,\{Q\} \qquad \{P \wedge {\sim}B\}\,C_2\,\{Q\}}{\{P\}\;\textbf{if } B \textbf{ then } C_1 \textbf{ else } C_2\,\{Q\}}$$

If-then rule, both ways

$$\frac{\{P \wedge B\}\,C_1\,\{Q\} \qquad P \wedge {\sim}B \Rightarrow Q}{\{P\}\;\textbf{if } B \textbf{ then } C_1\,\{Q\}}$$

While rule

$$\frac{\forall k.(\{I(k) \wedge B\}\,C\,\{I(k+1)\}) \qquad P \Rightarrow I(0) \qquad I(k) \wedge {\sim}B \Rightarrow Q \qquad \exists k.(I(k) \Rightarrow {\sim}B)}{\{P\}\;\textbf{while } B \textbf{ do } C\,\{Q\}}$$

Problem 1. In each of parts (a)–(e), find the weakest precondition P that makes the given Hoare triple correct. Simplify your answers (logically and algebraically) where applicable.

(a) $\{P\}\,\mathtt{x} := \mathtt{5} - \mathtt{x}\,\{n = x + 7\}$

(b) $\{P\}\,\mathtt{a} := -\mathtt{a};\ \mathtt{n} := \mathtt{n} + \mathtt{a}\,\{n \neq a \wedge n \neq 0\}$

(c) $\{P\}\,\mathtt{a} := \mathtt{a} + \mathtt{b};\ \mathtt{b} := \mathtt{a} - \mathtt{b};\ \mathtt{a} := \mathtt{a} - \mathtt{b}\,\{a \geqslant b + 2\}$

(d) $\{P\}\,\textbf{if } \mathtt{x} > \mathtt{y} \textbf{ then } \mathtt{x} := \mathtt{x} + 2 \textbf{ else } \mathtt{y} := \mathtt{y} - 2\,\{x \neq y\}$

(e) $\{P\}\,\textbf{if } \mathtt{x} > \mathtt{y} \textbf{ then } \mathtt{z} := \mathtt{x} \textbf{ else } \mathtt{z} := \mathtt{y}\,\{z > y\}$