MATH/CSCI 2112, DISCRETE STRUCTURES I, FALL 2005

Natural Deduction worked examples (from Handout 2).

Handout 2, #11 There are two directions in this problem. First, we prove $A \wedge (A \vee B) \vdash A$:

1	$A \wedge (A \vee B)$	
2	A	$\wedge$E, 1

Then, we prove $A \vdash A \wedge (A \vee B)$:

1	A	
2	$A \vee B$	$\vee$I, 1
3	$A \wedge (A \vee B)$	$\wedge$I, 1, 2

Handout 2, #15 There are 2 directions. First $A \to (B \wedge C) \vdash (A \to B) \wedge (A \to C)$:

1	$A \to (B \wedge C)$	
2	A	
3	$B \wedge C$	$\Rightarrow$E, 1, 2
4	B	$\wedge$E, 3
5	$A \to B$	$\Rightarrow$I, 2–4
6	A	
7	$B \wedge C$	$\Rightarrow$E, 1, 6
8	C	$\wedge$E, 7
9	$A \to C$	$\Rightarrow$I, 6–8
10	$(A \to B) \wedge (A \to C)$	$\wedge$I, 5, 9

The other direction is $(A \to B) \wedge (A \to C) \vdash A \to (B \wedge C)$:

1	$(A \to B) \wedge (A \to C)$	
2	A	
3	$A \to B$	$\wedge$E, 1
4	B	$\Rightarrow$E, 2, 3
5	$A \to C$	$\wedge$E, 1
6	C	$\Rightarrow$E, 2, 5
7	$B \wedge C$	$\wedge$I, 4, 6
8	$A \to (B \wedge C)$	$\Rightarrow$I, 2–7

Handout 2, #22

1	$\sim A \to \sim B$	
2	B	
3	$A \to \sim B$	
4	A	
5	$\sim B$	$\Rightarrow$I, 3, 4
6	B	R, 2
7	$\perp$	$\neg$E, 5, 6
8	$\sim A$	$\neg$I, 4–7
9	$\sim B$	$\Rightarrow$I, 1, 8
10	$\perp$	$\neg$E, 2, 9
11	$\sim(A \to \sim B)$	$\neg$I, 3–10
12	$B \to \sim(A \to \sim B)$	$\Rightarrow$I, 2–11

Handout 2, #24

1	$\vert\ A \vee B$	
2	$\vert\ \vert\ B \rightarrow A$	
3	$\vert\ \vert\ \vert\ A$	
4	$\vert\ \vert\ \vert\ A$	R, 3
5	$\vert\ \vert\ A \rightarrow A$	$\Rightarrow$I, 3–4
6	$\vert\ \vert\ A$	$\vee$E, 1, 2, 5
7	$\vert\ (B \rightarrow A) \rightarrow A$	$\Rightarrow$I, 2–6

Handout 2, #32

1	$\vert\ (A \rightarrow B) \vee (A \rightarrow C)$	
2	$\vert\ \vert\ A$	
3	$\vert\ \vert\ \vert\ A \rightarrow B$	
4	$\vert\ \vert\ \vert\ B$	$\Rightarrow$E, 2, 3
5	$\vert\ \vert\ \vert\ B \vee C$	$\vee$I, 4
6	$\vert\ \vert\ (A \rightarrow B) \rightarrow (B \vee C)$	$\Rightarrow$I, 3–5
7	$\vert\ \vert\ \vert\ A \rightarrow C$	
8	$\vert\ \vert\ \vert\ C$	$\Rightarrow$E, 2, 7
9	$\vert\ \vert\ \vert\ B \vee C$	$\vee$I, 8
10	$\vert\ \vert\ (A \rightarrow C) \rightarrow (B \vee C)$	$\Rightarrow$I, 7–9
11	$\vert\ \vert\ B \vee C$	$\vee$E, 1, 6, 10
12	$\vert\ A \rightarrow (B \vee C)$	$\Rightarrow$I, 2–11

Handout 2, #44

1	$\vert\ \sim B \rightarrow \sim A$	
2	$\vert\ \vert\ A$	
3	$\vert\ \vert\ \vert\ \sim B$	
4	$\vert\ \vert\ \vert\ \sim A$	$\Rightarrow$E, 1, 3
5	$\vert\ \vert\ \vert\ \bot$	$\neg$E, 2, 4
6	$\vert\ \vert\ \sim(\sim B)$	$\neg$I, 3–5
7	$\vert\ \vert\ B$	$\neg\neg$E, 6
8	$\vert\ A \rightarrow B$	$\Rightarrow$I, 2–7

Handout 2, #48

1	$\vert\ \sim A \rightarrow B$	
2	$\vert\ \vert\ \sim(A \vee (\sim A \wedge B))$	
3	$\vert\ \vert\ \vert\ A$	
4	$\vert\ \vert\ \vert\ A \vee (\sim A \wedge B)$	$\vee$I, 3
5	$\vert\ \vert\ \vert\ \bot$	$\neg$E, 2, 4
6	$\vert\ \vert\ \sim A$	$\neg$I, 3–5
7	$\vert\ \vert\ B$	$\Rightarrow$E, 1, 6
8	$\vert\ \vert\ \sim A \wedge B$	$\wedge$I, 6, 7
9	$\vert\ \vert\ A \vee (\sim A \wedge B)$	$\vee$I, 8
10	$\vert\ \vert\ \bot$	$\neg$E, 2, 9
11	$\vert\ \sim\sim(A \vee (\sim A \wedge B))$	$\neg$I, 2–10
12	$\vert\ A \vee (\sim A \wedge B)$	$\neg\neg$E, 11