MATH/CSCI 2113, DISCRETE STRUCTURES II, Winter 2010

Handout 3: Conditional Probability
Friday, February 3, 2010

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Problem 1. Consider two random events A and B. You are given the following information: $P(A) = 0.3$, $P(B) = 0.4$, $P(A|B) = 0.25$. (a) Calculate $P(\bar{A})$. (b) Calculate $P(A \cap B)$. (c) Calculate $P(\bar{A} \cap B)$. (d) Calculate $P(\bar{A}|B)$. (e) Calculate $P(B|A)$. (f) Are A and B independent?

Problem 2. You have three colored cards. One card is red on both sides. One card is red on one side and green on the other. One card is red on one side and blue on the other. A dealer shuffles the cards and places a random card on the table. The visible side of the card is red. What is the probability that if you flip the card, it will be green on the other side? Hint: make sure you use the correct event space where all possibilities are equally likely.

Problem 3. There are two identical-looking urns. Urn A contains 3 red balls and 5 blue balls. Urn B contains 3 red balls and 2 blue balls. (a) You pick an urn at random, and then you pick a ball at random from that urn. What is the probability that you get a red ball? (b) Your friend picks an urn at random, and then a ball at random. The ball she picked is blue. What is the probability it came from Urn A?

Problem 4. By analyzing several million spam and non-spam emails, a spam filter has determined that a random spam email has a 30 percent chance of containing the word "money", whereas a random non-spam email has a 5 percent chance of containing the word "money". It has further been determined that approximately 40 percent of all incoming emails are spam. (a) According to this data, what is the probability of a random incoming message being spam, if it contains the word "money"? (b) In the absence of other evidence, what is the probability of a random email being spam if it does not contain the word "money"?

Problem 5. There are three identical-looking urns. Urn A contains 25 red balls and 5 blue balls. Urn B contains 15 red balls and 15 blue balls. Urn

C contains 10 red balls, 10 blue balls, and 10 green balls. (a) You pick an urn at random, and then you pick a ball at random from that urn. What is the probability that you get a red ball? (b) Your friend picks an urn at random, and then a ball at random. The ball she picked is red. What is the probability it came from Urn A? (c) Your friend picks a random ball from a random urn, and the ball is blue. She puts the ball back in the urn that it came from, and hands that urn to you. You pick a random ball from the urn. With what probability do you pick a green ball?