MATH/CSCI 2113, DISCRETE STRUCTURES II, Winter 2010

Handout 5: Last year's midterm
Prof. P. Selinger

This is a "closed book" test. No notes or papers are allowed. You have 50 minutes. When possible, state integer answers as a decimal integer, and rational answers as a fraction.

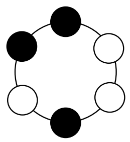
Problem 1 (5 points). How many subsets of $\{1, 2, \ldots, 12\}$ contain exactly twice as many even as odd numbers?

Problem 2 (5 points). n fair dice are rolled. What is the probability that the highest number rolled is 4?

Problem 3 (5 points). How many 4-digit numbers are there with the digits in strictly decreasing order? Examples: 7521, 9840, 3210 are allowed, 8554, 5211, 2130 are not allowed.

Problem 4 (5 points). There are two identical-looking urns. Urn A contains 2 red balls and 8 blue balls. Urn B contains 3 red balls and 2 blue balls. (a) You pick an urn at random, and then you pick a ball at random from that urn. What is the probability that you get a red ball? (b) Your friend picks an urn at random, and then a ball at random. The ball she picked is blue. What is the probability it came from Urn A?

Problem 5 (5 points). Consider circular necklaces made up of 6 colored beads. If there are 2 colors available, then there are $2^6 = 64$ such necklaces. Here is an example:



We call two necklaces equivalent if they differ by a rotation. How many equivalence classes are there?

Note: this year's midterm (2010) will also cover recurrences. See Handout 4 problems 1,2,4,5 for sample problems. Generating functions are not covered.