Propositional Logic

Propositions are made from propositional symbols, such as p, q, r, and propositional connectives such as "and", "or", "not", and "implies".

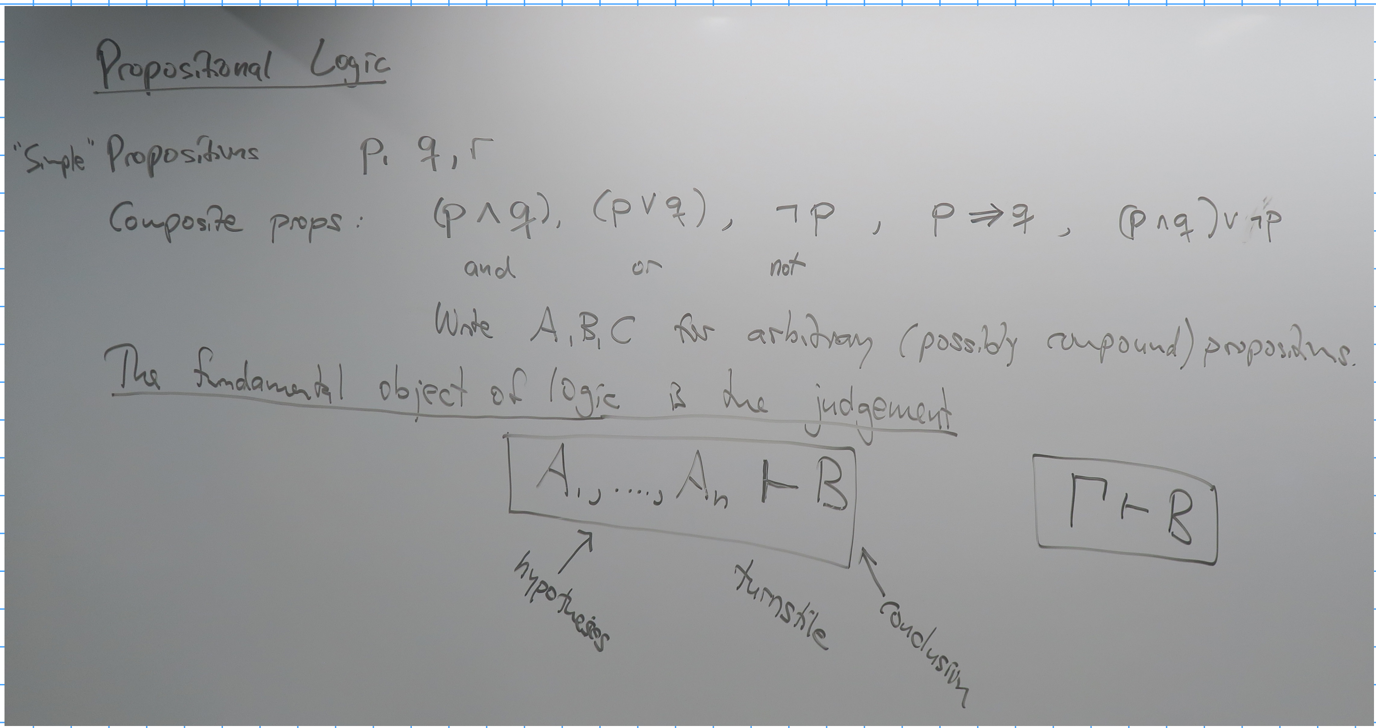
Write A, B, C for arbitrary propositions, e.g.: $A = (p \wedge q) \Rightarrow \neg p$.

A list of propositions is called a <u>context:</u> $\Gamma = A_1, \ldots, A_n$

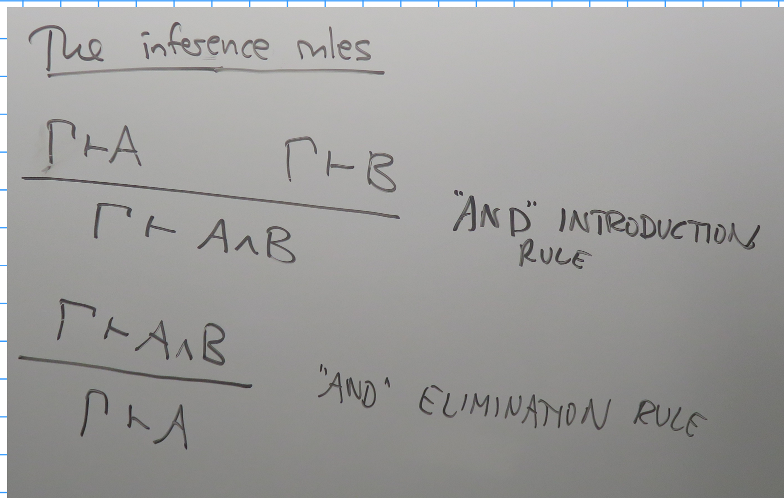
The fundamental object of logic is the <u>judgement:</u> $A_1, \ldots, A_n \vdash B$.
B is called the <u>conclusion</u>, and $A_1, \ldots, A_n$ are called the <u>hypotheses.</u>

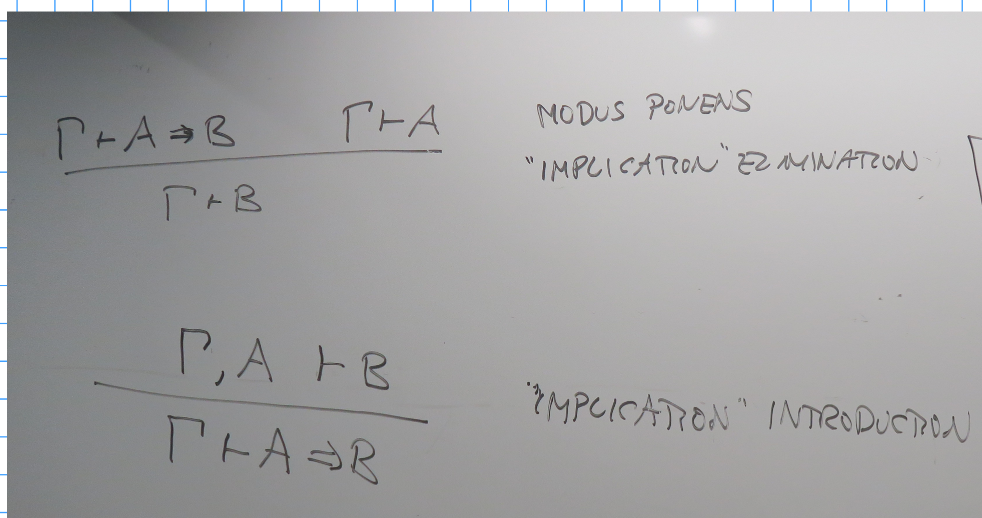
The judgement is <u>valid</u> if the hypotheses imply the conclusion.

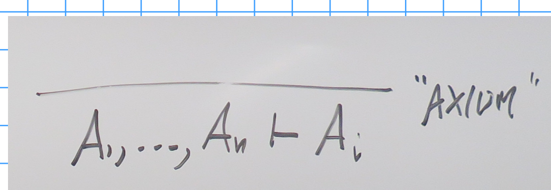
Note: the purpose of logic is not to determine which propositions are true. Its purpose is to determine which propositions <u>follow from other propositions.</u>



Valid judgements can be derived by a small number of inference rules. (There are approximately 13 of them, depending how you count). Here are some examples.







Here is an example of a derivation (or proof). We prove a judgement by repeatedly applying inference rules. This particular derivation proves that the judgement $\vdash p \wedge q \Rightarrow q \wedge p$ is valid. In other words, that the proposition $p \wedge q \Rightarrow q \wedge p$ follows from an empty set of hypotheses.