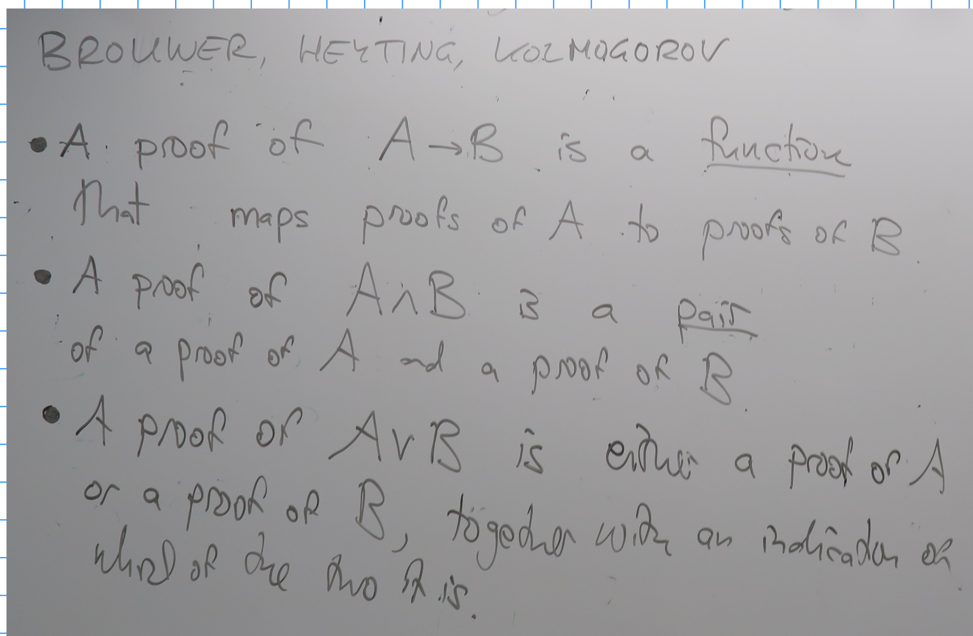
Lecture notes, Sept 22, 2026

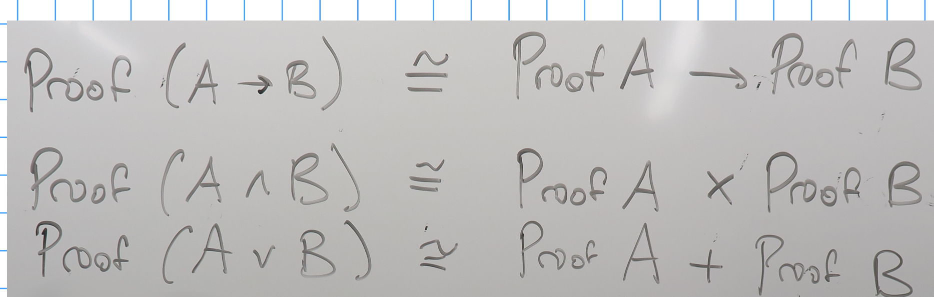
Recall the Brouwer-Heyting-Kolmogorov interpretation of logic. We saw the cases for implication and "and" last time. Today, we also added "or".



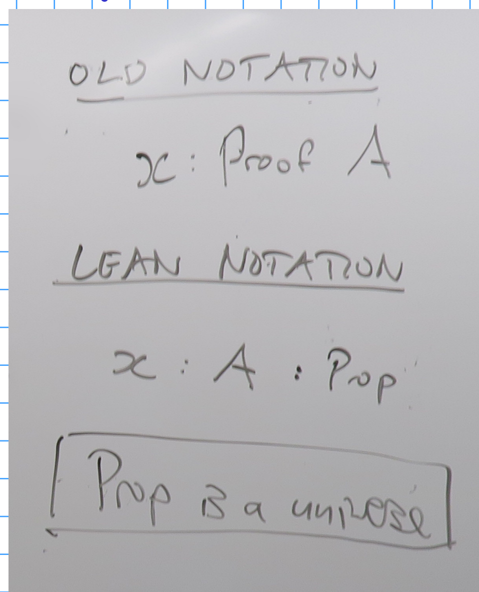
We write $X + Y$ for the disjoint union of two sets X and Y. The disjoint union means: rename all the elements of X and of Y so that they become distinct. Then take the union. For concreteness, we "rename" the elements of X as "left x", and the elements of Y as "right y". So:

$$X + Y = \{\text{left } x \mid x \in X\} \cup \{\text{right } y \mid y \in Y\}.$$

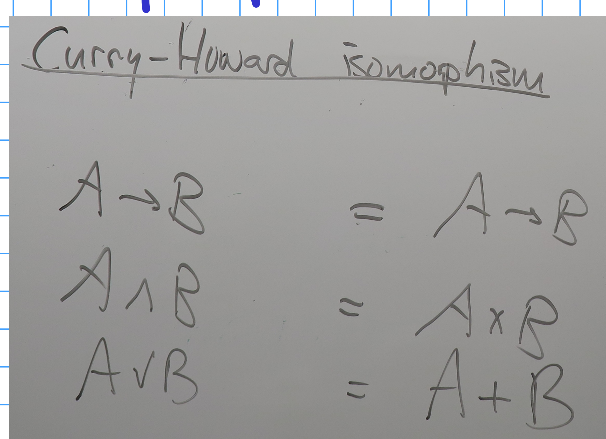
With these conventions, we can summarize the BHK interpretation in symbols:



The Curry-Howard correspondence (or Curry-Howard isomorphism) goes one step further, and removes the word "Proof". Instead of saying that "Proof A" is the set of proofs of A, we say that the proposition A itself is the set of its proofs. So a proposition A is a type, and its elements are the proofs of A. Since the elements of Prop are types, Prop is a universe.



Under the Curry-Howard isomorphism, the BHK correspondence then becomes particularly simple:



The slogans are: propositions are types, proofs are terms.
We also say that "proofs are programs".
This only works for "constructive" proofs. The BHK interpretation does not admit the excluded middle principle $A \vee \neg A$.