MATH 285, HONORS MULTIVARIABLE CALCULUS, FALL 1999

Problem Set 3

Worksheet: Determinants in 3 dimensions (in class)

Let $\vec{a} = \langle a_1, a_2, a_3 \rangle$, $\vec{b} = \langle b_1, b_2, b_3 \rangle$, and $\vec{c} = \langle c_1, c_2, c_3 \rangle$ be three 3-dimensional vectors. The determinant of $\vec{a}$, $\vec{b}$, and $\vec{c}$ is written as $\det(\vec{a}, \vec{b}, \vec{c})$ or

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$

It is defined as

$$\det(\vec{a}, \vec{b}, \vec{c}) = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}.$$

Problem 1 Calculate

$$\begin{vmatrix} 1 & 3 & 3 \\ 0 & 4 & -2 \\ 2 & 2 & 1 \end{vmatrix}.$$

Problem 2 Write down a formula for $\det(\vec{a}, \vec{b}, \vec{c})$ that does not use 2-dimensional determinants.

Problem 3 Prove that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = a_1 b_2 c_3.$$

Problem 4 Prove that the determinant satisfies the following properties:

1. $\det(\vec{a}, \vec{b}, \vec{c}) = \det(\vec{b}, \vec{c}, \vec{a}) = \det(\vec{c}, \vec{a}, \vec{b})$, i.e., when *rotating* the rows, the determinant does not change.
2. $\det(\vec{a}, \vec{b}, \vec{c}) = -\det(\vec{b}, \vec{a}, \vec{c})$, i.e., when *swapping* two rows, the determinant changes signs.
3. $\det(\lambda\vec{a}, \vec{b}, \vec{c}) = \det(\vec{a}, \lambda\vec{b}, \vec{c}) = \det(\vec{a}, \vec{b}, \lambda\vec{c}) = \lambda \det(\vec{a}, \vec{b}, \vec{c})$, where λ is a scalar.
4. $\det(\vec{a} + \vec{a}', \vec{b}, \vec{c}) = \det(\vec{a}, \vec{b}, \vec{c}) + \det(\vec{a}', \vec{b}, \vec{c})$.

Problem 5 Conclude from the above properties that the determinant does not change when a multiple of one row is added to another row. In symbols, $\det(\vec{a}, \vec{b}, \vec{c}) = \det(\vec{a}, \vec{b} + \lambda\vec{a}, \vec{c})$. Use this fact, together with Problem 3, to find a quicker way of calculating the determinant in Problem 1. (Hint: use a form of Gaussian elimination).

Homework Problems (due Tuesday 10/5)

13.1 #12, 16, 40; 13.2 #30, 32, 34 (see Example 5, p. 828); 13.3 #58, 61, 62;