MATH 2051, Problems in Geometry
Fall 2007
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Homework Sheet 2
Due in: Wednesday 26th September, 11:30 PM

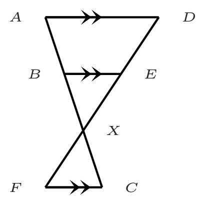
Note that diagrams are not drawn to scale. Scale drawing does **not** constitute a proof. Justify all your answers.

Compulsory questions

1 Suppose $ABCD$ is a parallellogram (so AB is parallel to DC and BC is parallel to AD) and E is the midpoint of BC, we draw the line AE, and let X be the point where AE intersects BD. Find the ratio $\frac{BX}{XD}$.

2 Suppose that lines l_1 and l_2 are parallel. Let A and B be points on l_1 and C and D be points on l_2 such that $AC = BD$, and the lines AC and BD intersect at a point X between l_1 and l_2. Show that $AD = BC$.

3 Let l_1, l_2 and l_3 be 3 parallel lines with l_2 between l_1 and l_3. Suppose we also have lines m and n which meet at X between l_2 and l_3. Call the points where m intersects l_1, l_2 and l_3 A, B and C respectively. Call the points where n meets l_1, l_2 and l_3, D, E and F respectively. Show that if $AB = XC$ then $AD = BE + CF$.



4 Let $ABCD$ be a quadrilateral. Let M, N, O, and P be the midpoints of AB, BC, CD and DA respectively. Show that $MNOP$ is a parallellogram.

5 Let A, B, C and D be four points going clockwise round a circle. Suppose AB and DC meet at X, nearer to B than A (and nearer to C than D). If $BX = 2$cm, $BD = 4$cm, and $AC = 5$cm, find CX.

6 Let l_1 and l_2 be parallel lines, and let Γ be a circle. Suppose l_1 meets Γ at A and B, and l_2 meets Γ at C and D. Show that $AC = BD$.

7 Let A, B and C be three points on a circle Γ, and let M be the point on the arc from A to B not containing C, such that $\angle ACM = \angle BCM$. Let O be the centre of Γ, and let OM meet CB at X. Show that O, C, X and A are concyclic (i.e. they all lie on a common circle).