MATH 2051, Problems in Geometry
Fall 2007
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Homework Sheet 3
Due in: Wednesday 3rd October, 11:30 PM

Note that diagrams are not drawn to scale. Scale drawing does **not** constitute a proof. Justify all your answers.

Compulsory questions

1 Let $ABCD$ be a cyclic quadrilateral. Let AB and DC meet at P (outside the circle, nearer to B than to A, and nearer to C than to D). Let AC and BD meet at X (inside the circle) and suppose that $PBXC$ is also cyclic. Prove that PX is a diameter of the circle through P, B, X and C.

2 Let X, Y, and Z be three points inside a circle Γ, and let the lines XY, YZ, and ZX meet Γ at: A and D; B and E; and C and F respectively. Suppose that $AY = DX$ and $BZ = EY$. Prove that $CZ = FX$.

3 Let circles Γ_1 and Γ_2 have centres O_1 and O_2 and radii r_1 and r_2 respectively. Suppose that Γ_1 and Γ_2 touch externally at a point S (i.e. they meet at S, and they have the same tangent there, and they are on opposite sides of the tangent). Suppose that the line l is tangent to Γ_1 at T and tangent to Γ_2 at U.

(a) Show that $\angle TSU = 90°$.

(b) Show that $TU^2 = 4r_1r_2$

4 Let Γ be a circle, centre O, and let P be a point outside Γ. Let l be a line through P, meeting Γ at A and B, and let PT be a tangent to Γ at T. Let the line TO meet AB at V, and let U be the point on AB such that $\angle TUV = 90°$. Show that $PU \times PV = PA \times PB$.

5 Let circles Γ_1 and Γ_2 have centres O_1 and O_2 and radii r_1 and r_2 respectively. Let them intersect at two distinct points B and C, and let A be a point on BC outside both circles. Let AT be a tangent to Γ_1, and let AU be a tangent to Γ_2, with T and U on Γ_1 and Γ_2 respectively. Let the lines TO_1 and UO_2 meet at V. Show that $TV = UV$.

6 Let Γ be a circle with centre O and radius r. Let P be a point outside Γ. Let T and T' be the points on Γ, such that the tangents to Γ at T and T' pass through P. Let X be the point of intersection of OP and TT'. Prove that $OP \times OX = r^2$.