MATH 2051, Problems in Geometry
Fall 2007
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Homework Sheet 4
Due in: Wednesday 10th October, 11:30 AM

Note that diagrams are not drawn to scale. Scale drawing does **not** constitute a proof. Justify all your answers.

Compulsory questions

1 Let ABC be an acute angled triangle. Let Γ be it's circumcircle. Let the tangents to Γ at B and C meet at a point P. Let M be the midpoint of BC. Show that $\frac{\sin \angle BAP}{\sin \angle PAC} = \frac{\sin \angle CAM}{\sin \angle MAB}$.

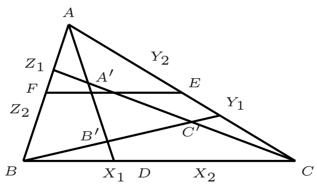
2 Let ABC be a triangle. Let the altitudes of the triangle from A, B, and C meet the circumcircle again at A', B', and C' respectively. Find the angles of triangle $A'B'C'$ in terms of those of ABC.

3 Let ABC be a triangle with orthocentre H. Show that if the triangles ABH, ACH and BCH have the same perimeter, i.e. if $AB+AH+BH = AC+AH+CH = BC+BH+CH$, then triangle ABC is equilateral (i.e. $AB = AC = BC$).

4 Let ABC be a triangle. Let the midpoints of the sides BC, AC and AB be L, M and N respectively. Let X_1 and X_2 be the points of trisection of BC, with X_1 nearer to B, i.e. $BX_1 = X_1X_2 = X_2C$. Similarly, let Y_1 and Y_2 be the points of trisection of AC, with Y_1 nearer to C, and let Z_1 and Z_2 be the points of trisection of AB, with Z_1 nearer to A.

(a) Prove that CZ_1 passes through the midpoint of FE.

Let A' be the point of intersection of AX_1 and CZ_1, B' the point of intersection of AX_1 and BY_1, and C' the point of intersection of BY_1 and CZ_1.



(b) Show that $Z_1A' = \frac{Z_1C}{7}$.

(c) Show that $A'C' = C'C$.

5 Let ABC be an acute angled triangle, and let P be a point in ABC. Show that

$$\frac{AP \times BC}{\sin(\angle BPC - \angle BAC)} = \frac{BP \times AC}{\sin(\angle APC - \angle ABC)} = \frac{CP \times AB}{\sin(\angle APB - \angle ACB)}$$

[Hint: Let D, E and F be the feet of the perpendiculars from P to BC, AC and AB respectively. Use the sine rule.]