MATH 2051, Problems in Geometry
Fall 2007
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Homework Sheet 6
Due in: Wednesday 7th November, 11:30 AM

Note that diagrams are not drawn to scale. Scale drawing does **not** constitute a proof. Justify all your answers.

Compulsory questions

1 Given a circle Γ, find a straight-edge and compass construction for its centre, and show that your construction does give the centre of Γ.

2 Given a circle Γ, and a point P outside Γ, find a straight-edge and compass construction for the points T_1 and T_2 on Γ such that PT_1 and PT_2 are tangent to Γ.

3 There is a convex semiregular polyhedron with one pentagonal face and two hexagonal faces meeting at each vertex. How many

i pentagonal faces

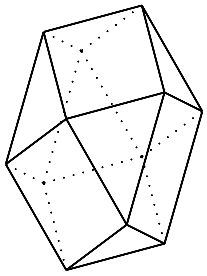
ii hexagonal faces

iii edges

iv vertices

does it have?

4 Recall that a cubeoctahedron is a semiregular polyhedron with 8 triangular faces and 6 square faces, with 2 triangular faces and 2 square faces meeting at each vertex. It's vertices are the midpoints of the edges of a cube (or an octahedron).



(a) What shape are the faces of the dual of the cubeoctahedron? [Hint: they are not regular polygons. Think about the symmetries of the cubeoctahedron (which are the same as symmetries of a cube). You do not have to give all the angles of the shape, just the type of shape]

(b) How many faces does it have?

5 Given a convex polyhedron, and a vertex X, let D_X be $360°$ minus the sum of all the angles at X (so for a regular tetrahedron, $D_X = 180°$ for each vertex X, since there are 3 angles of $60°$ at each vertex). Prove that the sum of the D_X over all vertices is $720°$. (For example, for the regular tetrahedron, we have 4 vertices with $D_X = 180°$, so the total is $4 \times 180° = 720°$.) [Hint: the sum of the angles in a polygon with n sides is $180(n-2)°$.]

6 In the same way that we constructed duals for semiregular 3-dimensional shapes, we can construct duals for semiregular 4-dimensional shapes. We place a vertex in the centre of each 3-dimensional hyperface, join two vertices by an edge if the 3-dimensional hyperfaces are adjacent, join a collection of vertices with a 2-dimensional face if the 3-dimensional hyperfaces all meet at an edge, and join a collection of vertices with a 3-dimensional hyperface if the corresponding 3-dimensional hyperfaces all meet at a vertex of the original shape.

What shape are the 3-dimensional hyperfaces of the dual of the hypercube?