MATH 2051, Problems in Geometry
Fall 2007
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Homework Sheet 8
Due in: Wednesday 21st November, 11:30 AM

Note that diagrams are not drawn to scale. Scale drawing does **not** constitute a proof. Justify all your answers.

Compulsory questions

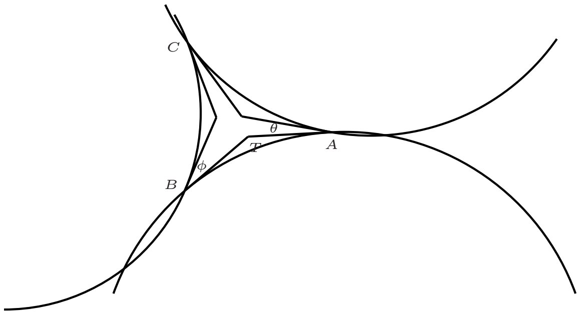
1 (a) We showed that an hyperbolic line is a Euclidean circle perpendicular to the boundary of the unit disc. Show that the origin has power 1 with respect to this circle.

(b) Show that given 3 (Euclidean) circles, Γ_1, Γ_2, and Γ_3, if the radical axis of Γ_1 and Γ_2 passes through the centre of Γ_3, and the radical axis of Γ_1 and Γ_3 passes through the centre of Γ_2, then the radical axis of Γ_2 and Γ_3 passes through the centre of Γ_1.

(c) Let ABC be an hyperbolic triangle such that the (hyperbolic) perpendiculars from A to BC and from B to AC meet at X. Show that the hyperbolic line from C to X is perpendicular to AB. (So X is the orthocentre of the hyperbolic triangle ABC.)

2 (a) Hyperbolic circles in the disc model are actually Euclidean circles, and the line joining the hyperbolic centre and the Euclidean centre passes through the origin. Let l be an hyperbolic line not passing through the origin, and A and B be points on l. l is a Euclidean circle; let the (Euclidean) tangents to this circle meet at T. Show that the midpoint of AB is the point where the line from the origin through T meets l. [Hint: consider the hyperbolic circle with diameter AB - it meets l perpendicularly at A and B.]

(b) Let Γ_1, Γ_2 and Γ_3 be (Euclidean) circles with centres O_1, O_2 and O_3, and radii r_1, r_2, and r_3 respectively. Let Γ_1 and Γ_2 meet at C, Γ_1 and Γ_2 meet at B, and Γ_2 and Γ_3 meet at A. Let T be the point where the tangents to Γ_3 at A and B meet. Suppose that T lies on the radical axis of Γ_1 and Γ_2. Let the angle between Γ_1 and Γ_2 at C (i.e. the angle between their tangents at C) be ψ, the angle between Γ_1 and Γ_3 at B be ϕ, and the angle between Γ_2 and Γ_3 at A be θ. Show that $r_1 \sin\phi = r_2 \sin\theta$.



[Hint: Let the tangent to Γ_3 from A meet Γ_2 again at A', and let the tangent to Γ_3 from B meet Γ_1 again at B'. Show that $AA' = BB'$.]

(c) Deduce that if ABC is an hyperbolic triangle, L, M and N are the midpoints of the sides BC, AC and AB respectively, and AL and BM meet at X, then the (hyperbolic) line CX passes through N.