MATH 2112/CSCI 2112, Discrete Structures I
Winter 2007
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Homework Sheet 3
Due in: Wednesday 31st January, 1:30 PM

Compulsory questions

1 Prove or disprove the following directly from the definitions:

(a) For any odd positive integer n, at least one of the numbers n, $n+2$, $n+4$, and $n+6$ is composite.

(b) 113 is prime.

(c) 142 is even

(d) For any integer n, if n is even, then so is $n^3+7n+12$.

(e) There is a natural number n such that $n^2+7n+12$ is prime.

2 Prove or disprove the following. You may use the results proved in the course, or in earlier questions. You do not need to write out proofs that particular numbers are prime.

(a) If x is a rational number, and there is an integer n such that nx is an integer, then x must be an integer.

(b) There are natural numbers m and n such that m, n, and $m+n$ are all prime.

(c) If x and y are rational numbers, and n is an integer such that nx is an integer, and ny is an integer, then $n^2(x^2+y)$ is an integer.

(d) All numbers of the form $12k+5$, where k is a natural number between 0 and 4, are prime.

(e) If $n>2$ is a positive integer, then at most 3 of $10n$, $10n+1$, ..., $10n+9$ are prime.

(f) There is a positive integer n such that n, $n+1$, $n+2$, ..., $n+100$ are all composite.