MATH 2600/STAT 2600, Theory of Interest
FALL 2013
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Sample Midterm Examination
Time allowed 1 hour 20 minutes

1. Mr. Almon receives an invoice for \$4,000, for payment within 50 days. He can get a 2% discount if he pays within the first 10 days. What is the largest rate of simple interest at which it would be worth his taking out a loan to get the discount.

2. Dr. Baker buys a promissory note for \$6,000 in 150 days at 3% simple interest. After 80 days, she sells it to a bank, which discounts notes at 4% simple interest.

   (a) How much does the bank pay for the note?

   (b) What is Dr. Baker's rate of return?

3. What rate of simple discount is equivalent to 5% simple interest over a period of 2 months?

4. Mr and Mrs. Carson are saving up for their children's education. They have three children, aged 7, 10 and 12. They invested \$50,000 two years ago, and \$30,000 today at $j_{12} = 5\%$ interest, and they want to divide this equally among their three children: when each child is 18 (at exactly this time of year), they will receive their share X. How much does each child get?

5. Which of the following interest rates is best for the lender?

   (i) 9% compounded quarterly

   (ii) 9.2% compounded annually

   (ii) 8.9% compounded monthly

6. What annual effective rate is equivalent to continuous compounding (constant force of interest) at 5%?

7. If force of interest is given by $\delta_t = 0.1 + 0.3t - 0.1e^t$ over a one-year period, how much needs to be invested at the start of the period, to cover a bill for \$8,000 at the end of the year?

8. The stock of company ABC currently pays a dividend of \$0.30 every quarter. Every quarter the company increases the dividend by 1%. The current price for the stock (just after a dividend of \$0.30 is payed) is \$15. What interest rate is being used to value this stock?

9. Mrs. Drake makes a loan of \$30,000 at $j_{12} = 7\%$. The loan is repaid over 6 years with equal monthly payments. When Mrs. Drake receives each payment, she immediately deposits it in an account which receives $j_{12} = 4\%$ interest. What yield does she make on her investment at the end of the 6 years?

10. A company buys a machine for \$40,000. The machine is expected to last for 4 years, after which it will have a salvage value of \$8,000. Prepare a depreciation schedule using:

    (a) The sum of digits method.

    (b) The constant percentage method

    (c) The straight line method

    (d) The compound interest method, with cost of capital $j_1 = 4\%$.

11. A company are deciding between two machines. The first machine costs \$130,000, lasts for 8 years, after which it has a resale value of \$16,000, and has maintainance costs of \$4,000 every year. The second machine costs \$220,000, lasts for 9 years, with a resale value of \$22,000, and has fuel and maintainance costs of \$3,000 in the first year, and increasing by \$80 in each subsequent year.

    (a) If the cost of capital is $j_1 = 8\%$, which machine has lower total capitalised cost?

12. Mr. Eccles takes out a loan for \$8,000, to be repayed over 24 months at $j_{12} = 8\%$. Calculate the outstanding balance after 5 months using:

    (a) The retrospective form.

    (b) The prospective form.

13. A loan of \$120,000 at $j_1 = 8\%$ is amortised with equal annual payments for 5 years.

    (a) Calculate the annual payments.

    (b) Draw up a complete amortisation schedule for the loan.

14. Mrs. Finch takes out a 25-year mortgage for a loan of \$200,000 at $j_2 = 7\%$.

    (a) Calculate the monthly payments required.

    (b) After 5 years, the interest rate rises to $j_2 = 9\%$, calculate the new monthly payments if she wishes to keep the mortgage over 25 years.

    (c) If instead, she wishes to keep the mortgage payments the same, when will she finish paying off the mortgage?

15. Mr. and Mrs. Green buy a cottage, with a downpayment of \$50,000 and a 15-year mortgage for the remaining \$150,000 at $j_2 = 5\%$. There is a penalty of three times monthly interest on the outstanding balance for paying off the loan early. After 3 years, another company offers them a

chance to refinance at $j_2 = 4.4\%$ for the remaining 12 years of the loan. Should they refinance?

16. Mrs. Horton buys a house in the US. She needs to borrow \$300,000 at $j_{12} = 7.2\%$, amortised over 15 years. There is also a financing fee of \$5,000. What is the APR for this loan?

17. A bank lends \$200,000 to Mr. and Mrs. Inglis. The loan is payed back with monthly interest-only payments at $j_{12} = 4\%$, with the principal returned as a lump sum after 15 years. After 8 years, the bank sells the loan to a private investor, who wishes to achieve an annual effective yield of 5.4%. How much does the investor pay for the loan?

18. Mrs. Jeeves borrows \$6,000 for one year at 7% simple interest. After 3 months, she repays \$3,000.

    If the loan is calculate using the merchant's rule, how much does she need to pay 8 months after the start of the loan, to pay off the debt?

19. Dr. Kearns borrows \$600,000 at 6% simple interest for one year. The US rule is used to calculate the outstanding balance. After 4 months, he has \$60,000. He can earn simple interest at 3% on this money. When should he repay this money in order to minimise the outstanding balance at the end of the year?

Formulae

$$s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$$

$$a_{\overline{n}|i} = \frac{1 - (1+i)^{-n}}{i}$$

$$(Ia)_{\overline{n}|i} = \frac{(1+i)^{-1} a_{\overline{n}|i} - n(1+i)^{-n}}{i}$$