MATH/STAT 2600, Theory of Interest
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In Class Examples

Compound Interest

Question 1

(a) Calculate the accumulated value on maturity of $5,000 invested for 17 years at 5% effective annual interest.

(b) How long would it take for the $5,000 to grow to $40,000 at this 5% annual interest rate?

(c) If the investor needs the $5,000 to grow to $40,000 after 26 years, what annual interest rate is needed?

Compound Interest

Source: www.xkcd.com

Compound Interest

Question 2

A man takes out a 10 year loan for $20,000 at 7% interest, compounded annually. After 4 years, the interest rate increases to 9%. He repays $12,000 after 5 years, then after 7 years from the start of the loan, he repays a further $4,000. Calculate the outstanding balance at the end of the 10 years.

Simple Interest

Question 3

Calculate the accumulated value on maturity of the following investments:
(a) $1,000 invested for 3 months at 5% simple interest.
(b) $200 invested on 16th August 2007 and maturing on 4th March 2008, at 7% simple interest. (NB. 2008 is a leap year.)

Simple Interest

Question 4

A man invests $1,000 at 12% simple interest on 7th March.
(a) On what date does he have enough money to buy a computer which costs $1,100 (including taxes)?
(b) How much money does he have left over if he buys the computer on that day?

Question 5

A businessman receives an invoice for $700, for payment within 50 days. The invoice also offers a 2% discount if payment is received within 10 days. What is the largest rate of simple interest at which it is worth his taking out a loan to get the discount?

Question 6

An investor has a promissory note for $10,000 in 240 days at 8% simple interest. After 230 days, she sells it to a bank which discounts notes at 7.8% interest.
(a) How much does the bank pay for the note?
(b) What rate of return does the investor achieve?

Present Value

Question 7

Mr. and Mrs. Philips are saving up to buy a car for their daughter's 18th birthday in 6 years time. The cost of the car is $25,000. If the annual effective interest rate is 4%, how much money do they need to invest now to be able to buy the car after 6 years?

Present Value

Question 8

What price should you pay for a T-bill of face value $10,000 maturing in 86 days, at a simple interest rate of 7%?

Question 9

Mr. Smith invests \$10,000 at 5% interest, compounded annually. He plans to use the investment to pay a bill of \$4,000 after 7 years. What is the largest amount *x* that he can take out after 1 year, and still have enough to pay the \$4,000 after 7 years?

Question 10

Mr. Smith takes out a loan of $10,000 at 5% simple interest on 11th May 2011. He pays it back with a repayment x on 12th July 2011, and a repayment of $4,000 on 3rd September 2011.

(a) Write down and solve an equation of value for x with focal date:

(i) 11th May.

(ii) 12th July.

(iii) 3rd September.

(b) Which (if any) of these gives the correct value for x?

Question 11

A woman invests $30,000 at 5% interest compounded annually. She wants to use this money to give $20,000 to each of her children when they turn 18. She currently has 2 children aged 9 and 4, and she plans to have a third child. How long does she have to wait before having the third child, in order for the investment to be sufficient?

Question 12

Calculate the accumulated value on maturity of $1,000 invested for 23 years at 5% interest compounded:
(a) quarterly.
(b) monthly.

Question 13

Which of the following rates of interest is best for the investor:
(i) 10% compounded monthly
(ii) 10.5% compounded annually
(iii) 9.8% compounded daily

Question 14

An investor has a promissory note for $10,000 in 8 years at 4% interest compounded monthly. After 3 years, 7 months, she sells it to a bank which discounts notes at 3% interest (also compounded monthly).
(a) How much does the bank pay for the note?
(b) What annual effective rate of return does the investor achieve?
(c) If the investor is looking to make a return of 5.5% on her investment, by what time does she need to sell it to the bank?

Question 15

What rate of simple discount is equivalent to 11% simple interest over a period of 9 months?

Question 16

What rate of compound discount is equivalent to 6% compound interest?

Question 17

What annual effective rate is equivalent to continuous compounding at 5%?

Force of Interest

Question 18

What rate of continuous compounding (constant force of interest) is equivalent to annual effective rate of 4%?

Question 19

Suppose that force of interest is given by $\delta_t = 0.05(t - t^2)$ for a period of one year. What is the accumulated value of \$1,000 invested at the start of the year?

Question 20

A man wants to buy a car which costs $20,000 today (including all taxes) and whose price is linked to inflation of 6%. He currently has $17,000, which he plans to invest until he has enough to buy the car. How long does he have to wait before he can afford to buy the car: [Use the theoretical method for fractional interest periods.]
(a) if he can invest his money at 8% compounded annually?
(b) if he can invest his money at 10% compounded annually?

Question 21

Mr. Harris wants to buy a house. Today, he would need a downpayment of $150,000 to buy the house, but he only has $90,000. He invests this money at $j_{12} = 7\%$. However, the downpayment needed increases with inflation at a rate of $j_{12} = 6\%$. How long must he wait before he has saved up enough to make the downpayment?

Question 22

Ms. Jones invests $500 a month in her pension plan, which pays interest at $j_{12} = 6\%$. She makes her first investment on her 21st birthday. How much money does she have in the plan when she retires on her 65th birthday (assuming that she still makes the payment on that birthday)?

Question 23

A car dealership allows a customer to pay for a car with monthly payments of $200 for 5 years. If the company is charging 10% interest on these payments, what is the price of the car?

Level Payment Annuities

Question 24

Jack wants to start a pension plan for his retirement. He decides that after he retires, he will need $1,000 a month from his pension. His pension fund pays interest at $j_{12} = 5\%$.

(a) If he plans to have enough in his pension fund to last for 25 years, how much will he need at the time he retires (assuming the payments start one month after his retirement).

(b) He is currently 43. If he plans to retire at 65, i.e. exactly 22 years from today, how much does he need to contribute to the pension plan every month to acheive the target in part (a), assuming he makes his first payment today, and his last one in 22 years.

Level Payment Annuities

Question 25

Dr. Proctor sets up a savings account on 1st January 2001 to pay for his childrens' education. He pays $100 into the account each month, starting on 1st January 2001, until 1st June 2004, when he increases the payments to $150 a month. Then from 1st April 2009 onwards, he reduces the payments back to $100 each month. The interest rate for the account is determined each year by market conditions, and the interest rates are as given in the following table:

Year	2001	2002	2003	2004	2005	2006
Interest rate	5%	5%	4%	4%	5.5%	5.5%
Year	2007	2008	2009	2010	2011	2012
Interest rate	8%	4%	4%	5%	5%	5%

How much is in the account when he makes his last payment on 1st August 2012?

Question 26

Mr. MacDonald makes a donation of $1,000,000 to his old University. He requests that it be used to set up a scholarship fund which pays $20,000 a year to each of 3 students, and should last for 100 years. If the money is invested at 5% interest per year, how long does the university have to wait before it can make the first payment from the scholarship fund.

Question 27

John makes a deposit of \$2,000 at the end of every year into an account that pays interest at $j_{12} = 5\%$. How much is in the account at the end of 12 years?

Question 28

Mr. Wilson wants to save up $20,000 by the time his daughter goes to university in 9 years time. He plans to make monthly payments into an account that gives $j_4 = 2\%$ interest. What should the payment be each month? [Assume the account uses compound interest for partial interest periods.]

Question 29

Ms. King buys a house for $250,000. She makes a down payment of $50,000 and takes out a mortgage for the remaining $200,000. The interest rate on the mortgage is $j_2 = 10\%$.

(a) If she takes a 20 year mortgage, what are the monthly payments, and what is the concluding smaller payment?

(b) (i) If she can afford up to $2,500 a month, how many years should her mortgage be (it has to be a whole number of years)?

(ii) What would the monthly payments and the final payment be in this case?

(c) (i) If she can invest her money at $j_{12} = 5\%$, how much would the money left over from her $2,500 a month, when following the payment schedule in (a), be worth at the end of 20 years?

(ii) What if she follows schedule (b)?

Question 30

A man takes out a loan for $10,000 at an interest rate $j_4 = 10\%$. He agrees to repay it with a set of quarterly payments of $500.

(a) How many payments does he need to make?

(b) If he agrees to pay off the debt by making a larger than usual payment on the last payment, calculate the value of the last payment? [When the next payment would be less than $500, he pays off the entire loan, instead of just $500.]

(c) If instead, he agrees to make normal payments until the amount due is less than $500, what will be the value of his last payment?

Question 31

A woman takes out a mortgage for $100,000, to buy a house. The bank tells her that she can pay it off with monthly payments of $1,000 for 20 years. What nominal rate of interest (compounded monthly) are they charging her?

Question 32

How much money is needed to establish a scholarship fund that will pay out \$20,000 a year forever, if the money is invested at $j_4 = 6\%$?

Question 33

Andrew starts a savings account on 1st January, which pays $j_{12} = 4\%$ interest. He plans to make monthly deposits to the account for 25 years, starting on 1st January, until he retires. He starts by making a monthly deposit of \$200. Every year, on 1st January, his salary increases by 4%, and he therefore plans to increase his deposits by 4% at this time. If he keeps up this planned deposit schedule, how much will be in the account when he retires on 1st January in 25 years time (he does not make a deposit at this time)?

Question 34

Joe has a salary of $40,000 a year. Every year, he gets a 5% salary increase. He needs $35,000 a year for his living expenses, and he saves the rest in an account which pays $j_1 = 3\%$ interest. How much will be in the account after he makes his 16th deposit in 15 years time?

Question 35

The stock of company ABC currently pays a dividend of \$0.50 every quarter. Every year the company increases the dividend by 5% every year. What is a fair price for the stock at $j_4 = 12\%$?

Question 36

Mrs. Barry buys a bond with face value $5,000, coupon rate 8%, semi-annual and maturity 15 years for a price which yields a rate of return of $j_2 = 11\%$.

(a) How much does she pay for the bond?

(b) If she can re-invest the coupons at 7% compounded semi-annually, what is the overall yield she achieves at the end of 15 years?

Question 37

A computer is bought for $1500. It is expected to last for 3 years, after which it will have a value of $300. Prepare a depreciation schedule using

(a) The straight-line method.
(b) The constant percentage method.
(c) The sum of digits method.
(d) The compound interest method.

Question 38

A machine has a current cost of $40,000. The machine has an expected lifetime of 20 years. It cannot be resold when it is finished with. The maintainance costs are $700 in the first year, and increase by $500 in each subsequent year. The cost of capital is $j_1 = 6\%$.

(a) What is the total capitalised cost if the machine is replaced every 20 years?

(b) It turns out to be best to replace the machine every 15 years. What is the total capitalised cost in this case?

(c) Another type of machine for the same task has a current cost of $80,000, but it's price is expected to fall by 4% every year, as the technology improves. It's maintainance costs are $2,000 a year. It also lasts 20 years. Would this machine be cheaper in the long run? [Retraining costs prevent buying the cheaper machine first, and then changing to the other machine when it becomes cheaper.]

Question 39

A loan of \$4,000 at $j_{12} = 8\%$ is amortised with equal monthly payments for 6 months, with the first payment in one month.
(a) Calculate the monthly payments.
(b) Draw up a complete amortisation schedule for the loan

Question 40

Mr. Thompson takes out a 25 year mortgage for a loan of \$250,000 at $j_2 = 7\%$. After 5 years, the interest rate increases to 9%.

(a) Calculate the outstanding balance after 5 years.

(b) What should the new monthly payments be if he wishes to keep the term of the mortgage at 25 years?

(c) If instead, he wishes to keep the monthly payments the same (or as near as possible with the mortgage a whole number of years):

(i) what will the new term of the mortgage be, and what will the monthly payments be?

(ii) How long must he wait before the payments after the interest rate change have as large a principal component as the last payment before the interest rate change?

Question 41

Mr. and Mrs. Johnson buy a house for $500,000 with a downpayment of $100,000 and a 20 year mortgage for the remaining $400,000 at a rate $j_2 = 6\%$. This results in monthly payments of $2848.76. The mortgage allows them to pay the balance off early for a penalty of three times the monthly interest on the outstanding balance.
After 7 years, at what interest rate should they be willing to take out a loan in order to pay off the remaining balance (including the penalty)?

Question 42

Jim takes out a loan for $15,000, to be repayed over 36 months at $j_{12} = 6\%$. Calculate the outstanding balance after 4 months using:
(a) The retrospective form.
(b) The prospective form.

Question 43

Matthew takes out a mortgage for $300,000 for 20 years at $j_2 = 6\%$.

(a) Calculate the monthly repayments.

(b) The loan is due to be refinanced after 5 years, when Matthew will be allowed to make changes to his monthly payments. The bank charges a penalty of 3 months interest for refinancing before that time. After 1 year, Matthew gets a promotion, and can now afford payments of up to $3,000 a month. If he can earn interest of $j_{12} = 3\%$ on his savings, should he refinance the loan early?

Question 44

Mr. and Mrs. Young buy a house in the US. After the downpayment, they need to borrow $350,000. The bank offers them a 25 year mortgage at $j_{12} = 0.06$, with a financing fee of $4,000. (So the actual loan amount is $354,000.) What is the APR in this case?

Question 45

Mr. Robinson borrows \$1,000,000 at $j_{12} = 9\%$ to set up his business. He has two options for repaying the debt. He may either amortise the debt over 15 years, or he may pay off the interest each month, and set up a sinking fund to pay off the debt at the end of 15 years. The sinking fund will earn $j_{12} = 8.5\%$.

(a) Calculate the monthly payments needed for each of the options.

(b) Mr. Robinson expects that he will want to invest a further \$200,000 into his business after 6 years. If he is paying off the loan using a sinking fund, he will just use the money in the sinking fund to pay it off. If he has taken the amortised loan, then he will have to take out another loan at $j_{12} = 12\%$ to make this investment. Calculate the new monthly payments after 6 years in each case.

Question 46

A bank lends $200,000 to Mr. and Mrs. Hunt. The loan is payed back with semiannual interest-only payments for 20 years at $j_2 = 6\%$, then the principal is returned as a lump sum after the 20 years. After 5 years, the bank sells the loan to another bank, which values the loan at interest rate $j_2 = 6.5\%$.

(a) How much does the second bank pay for the loan?

(b) What is the first bank's rate of return?

Question 47

A man takes out a one-year loan for $10,000 at 7% simple interest. After 4 months, he repays $2,500. After 7 months from the start of the loan, he repays a further $4,000. Calculate the outstanding balance at the end of the year, using the Merchant's Rule.

Question 48

A man borrows $1,000,000 at 7% simple interest for one year. The US rule is used to calculate the outstanding balance.

(a) How much does he need to repay after 6 months in order to reduce the amount due at the end of the year? [i.e. so that the balance due after one year is less than $1,070,000.]

(b) After 6 months, he has $20,000. He can earn simple interest at 3% on this money. When should he repay this money in order to minimise the outstanding balance at the end of the year?

Question 49

Calculate the price that should be paid for each of the following bonds to obtain the desired yield:

(a) face value \$5,000, maturing at par in 15 years, coupon rate 8%, desired yield $j_2 = 7\%$.

(b) face value \$2,000, maturing at 110 in 5 years, coupon rate 6%, desired yield $j_2 = 6.5\%$.

Question 50

Calculate the yield from the following bond, using linear interpolation to get a more accurate answer. Face value $1,000, maturing at par in 5 years, coupon rate 4%, bought for $922

Question 51

Write out a complete bond amortisation schedule for a bond with face value \$2,000 with coupon rate 5%, maturing at 105 in 4 years, sold to an investor who wishes to receive a yield of 6%.

Question 52

A bond with face value $500, maturity date 1st January 2033 and coupon rate 5% is sold on 4th March 2011 to an investor who wishes a yield of 4.5%. Calculate
(a) The flat price.
(b) The quoted price.

Question 53

An investor buys a bond with face value $1,000, maturing at par in 15 years, coupon rate 5%, for a price to yield 4%. He also buys a strip bond with face value $2,000, maturing in 15 years, for a price to yield 4%.

(a) How much does he pay for each of these bonds?

(b) 2 years later, interest rates drop, and he sells both bonds to investors who wish to receive a yield of 3.2%. What prices does he receive for them?

Question 54

A company are considering a project. The project has the following expected cash flows (all amounts are calculated at the beginning of the year):

Year	0	1	2	3	4	5	6	7	8	9	10
Net Cash Flow (000)	-100	-10	30	40	40	40	25	40	40	40	-50

(a) What is the Net Present value of the project at $j_1 = 5\%$?
(b) What is the Net Present value of the project at $j_1 = 15\%$?
(c) What is the internal rate of return?

Question 55

A company are considering two projects. The projects have the following expected cash flows (all amounts are calculated at the beginning of the year):

Year	0	1	2	3	4	5	6	7	8
Project 1 Net Cash Flow (000)	-100	-10	30	40	40	40	40	40	-50
Project 2 Net Cash Flow (000)	-80	-20	10	10	20	40	80	100	-30

(a) Which project should they prefer if the cost of capital is $j_1 = 3\%$?
(b) Which project should they prefer if the cost of capital is $j_1 = 14\%$?

Question 56

Mr. Clarke has $30,000 in his investment fund at the start of the year. Three months later, the fund has increased to $32,000, and he invests a further $15,000, then another month later, the fund has grown to $51,000, and he invests $4,000. After another 3 months, (7 months from the start of the year), the fund now has $53,500 and he withdraws $25,000. At the end of the year, there is only $24,600 in the fund.
(a) Calculate the dollar-weighted return.
(b) Calculate the time-weighted return.

Question 57

Mrs. Hargreaves has an account which pays interest at $j_1 = 2\%$ on credit balances, and charges interest at $j_1 = 8\%$ on debit balances. She is considering borrowing \$15,000 from this account to make an investment which will return \$2,000 at the end of the first year, \$4,000 at the end of the second and third years, and \$10,000 at the end of the fifth year. She is also considering borrowing \$20,000 for an investment that would return \$2,000 after 1 year, \$11,500 after 4 years, \$14,000 after 5 years and \$1,000 after 25 years. How much money does she have in her account if she invests in:
(a) The first project?
(b) The second project?
(c) Both projects?

Question 58

The current term structure has the following yields on zero-coupon bonds:

Term(years)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
rate	4%	5%	5.5%	5.8%	6%	6%	5.8%	5.6%

How much should be paid for a \$100 face-value bond with semi-annual coupons, maturing at par in 4 years time, with:
(a) 3% annual coupon rate?
(b) 6% annual coupon rate?
(c) 10% annual coupon rate?

Question 59

The current term structure has the following yields on zero-coupon bonds:

Term(years)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
rate	4%	5%	5.5%	5.8%	6%	6%

What is the yield to maturity of a 5% semi-annual bond maturing in 3 years time?

Question 60

Term(years)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
rate	6%	6%	5.5%	5.2%	5%	4.8%

What is the yield to maturity of a 3% semi-annual bond maturing in 3 years time?

Question 61

The spot-rates for 1, 2 and 3 year strip bonds are 5%, 6% and 6.5% annually.

(a) What is the implied forward rate for a 2-year loan starting in 1 year's time?

(b) If the term structure in one year's time is the same, what is the rate of return on a 3-year zero-coupon bond bought now, and sold in one year's time?

Question 62

The spot-rates for 11, 12 and 13 year strip bonds are 5%, 6% and 6.5% annually. What is the implied forward rate for a 2-year loan starting in 11 year's time?

Question 63

The spot-rates for 1, 2 and 3 year strip bonds are 5.5%, 6.5% and 7% annually. You have the opportunity to borrow or lend money at these rates, and you also have the opportunity to arrange to borrow or lend money in two year's time for one year, at an annual rate of 7.5%. Construct an arbitrage possibility.

Question 64

Mr. Simpson has a loan for $1,000,000 at a rate of prime+2. He pays interest only on the loan, in annual payments. He makes a forward rate agreement with Mr. Dodd, so that the interest rate for the second year (starting one year from now) will be 5.4%. In one year's time, the prime rate is 3%. How much money changes hands, and in which direction?

Question 65

Borrower *A* can borrow at 5% on the fixed-rate market, or at prime+1% on the variable rate market. Borrower *B* can borrow at 4% on the fixed-rate market, or at prime+0.5% on the variable rate market. Arrange a transaction that will allow *A* to borrow on the fixed-rate market and *B* to borrow on the variable-rate market, so that they both achieve better rates than the rates above.

Question 66

The current term structure has the following yields on zero-coupon bonds:

Term(years)	1	2	3	4	5
rate	5%	5.5%	5.6%	6%	6%

Mr. Wright has a floating rate loan of $600,000, with annual interest-only payments. He wishes to exchange this for a fixed rate over the next 5 years (i.e. he wants to pay the same interest rate over the next 5 years). What should this rate be?

Question 67

The current term structure has the following yields on zero-coupon bonds:

Term(years)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
rate	4%	5%	5.5%	5.8%	6%	6%

What is the at-par yield of a 3-year bond.

Question 68

Calculate the Modified Duration and Macauley Duration of a 5 year bond with yield rate $j_2 = 3\%$ and semi-annual payments at coupon rate
(a) 2%
(b) 5%
(c) 15%

Question 69

Consider a portfolio consisting of two zero-coupon bonds, maturing in 5 years and 20 years, with face values $10,000 and $5,000 respectively and yield rates of 5% and 6.3% respectively.

(a) Find the yield rate and face value of a 8-year zero-coupon bond which has the same present value and modified duration as this portfolio.

(b) Suppose that the term structure flattens, so that the 5-year spot rate increases to 5.5% and the 20-year spot rate decreases to 6%. Calculate the new present values and durations of the portfolio.

Question 70

Find a single liability which is immunised by a bond with face value $1,000 with coupon rate 5%, for semi-annual coupons for 8 years, at a yield rate of $j_2 = 6\%$.

Question 71

Find the values of A_0 and A_{15} that fully immunise a single payment $L_8 = 200,000$ at a flat term structure with annual interest rate $i_0 = 0.1$.

Question 72

(a) Is it possible to find a single future payment received, which immunises the liabilities of a 30-year bond with face value $1,000,000 and coupon rate 5%, at a yield rate of 3%.

Question 73

The current term structure has the following yields on zero-coupon bonds:

Term(years)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
rate	4%	5%	5.5%	5.8%	6%	6%

Recall from Chapter 6 that the yield to maturity of a 5% semi-annual bond maturing in 3 years time is 5.973%.
(a) Calculate the modified duration of this bond, based on a parallel shift in the term structure.
(b) Calculate the modified duration based on a flat term structure at the yield rate.