MATH 2600/STAT 2600, Theory of Interest
FALL 2014
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Homework Sheet 1
Model Solutions

1. *Calculate the accumulated value on maturity of the following investments:*

   (a) \$8,000 invested for 18 years at 2.5% effective annual interest.

   $8000(1.025)^{18} = 12477.27$.

   (b) \$3,000 invested for 8 months at 5% simple interest.

   $3000\left(1 + 0.05 \times \frac{8}{12}\right) = 3100$

   (c) \$30,000 invested for 6 years at $j_{12} = 9\%$.

   $30000(1.0075)^{72} = 51376.58$

2. *Mrs. Adams takes out a loan for \$25,000 at 4% effective annual rate. After 3 years, the interest rate increases to 5%. She repays \$9,000 after 6 years, and \$12,000 after 8 years. What is the outstanding balance after 9 years?*

   After 3 years she owes $25000(1.04)^3 = 28121.60$. After a further 3 years she owes $28121.60(1.05)^3 = 32554.27$. She repays \$9,000, so she owes \$23,554.27, so after 8 years she owes $23554.27(1.05)^2 = 25968.58$, and she repays \$12,000, so she owes \$13,968.58. Finally afte 9 years, the outstanding debt has grown to $13968.58(1.05) = \$14,667.01$.

3. *Mrs. Bruce buys a promissory note for \$6,000 in 150 days at 3% simple interest. After 80 days, she sells it to a bank, which discounts notes at 3% simple interest.*

   (a) How much does the bank pay for the note?

   At maturity, the note pays $6000\left(1 + 0.03\frac{150}{365}\right) = 6073.97$. When the bank buys the note, there are 70 days to maturity, so it pays $\frac{6073.97}{1+0.03\frac{70}{365}} = 6039.23$.

   (b) What is Mrs. Bruce's rate of return?

   Mrs. Bruce makes \$39.23 on an investment of \$6,000 over a period of 80 days. Her rate of return is therefore given by $\frac{39.23}{6000} \times 36580 = 0.0298$, so her rate of return is 2.98%.

4. *Mr. Cranch invests \$2,000 at 6% effective annual interest.*

   (a) How many years does he have to wait before he has saved up \$3,800?

   $2000(1.06)^n = 3800$, so $1.06^n = \frac{3800}{2000} = 1.9$, so $n\log(1.06) = \log(1.9)$, meaning $n = \frac{\log(1.9)}{\log(1.06)} = 11$ years.

(b) If he needs his investment to grow to \$3,800 within 7 years, what rate of interest does he need to invest at?

He needs $2000(1+i)^7 = 3800$, so $(1+i)^7 = 1.9$, so $i = 1.9^{\frac{1}{7}} - 1 = 9.6\%$.

5. *Mr. Davis receives an invoice for \$11,200, for payment within 50 days. He can get a 2% discount if he pays within the first 10 days. What is the largest rate of simple interest at which it would be worth his taking out a loan to get the discount.*

   If he pays the invoice on the last day, he pays \$11,200 after 50 days. If he wants to take advantage of the discount, he pays $11200(0.98) = 10976$ after 10 days. To do this, he needs to borrow \$10,976 for 40 days. The amount he has to repay after 50 days is therefore $10976\,(1 + ifrac40365)$, so for this to be worthwhile he needs $\frac{40\times 10976}{365} i < 224$, or $i < \frac{1024\times 365}{40\times 10976} = 0.1862$, so the largest rate at which it would be worth taking out the loan is 18.62%.

6. *Mr and Mrs. Ellis are saving up to buy cars for their children's 18th birthdays. They have two children, aged 3 and 9. They invest \$20,000 today at $j_{12} = 3\%$ interest, and want to use this to pay \$25,000 for each child's car, when the child is 18 (at exactly this time of year). How much more money do they need to add to this fund in two years time, in order to have enough?*

   We use an equation of value. Using today as the focal date, we have $20000 + X(1.0025)^{-24} = 25000(1.0025)^{-108} + 25000(1.0025)^{-180}$, or $20000 + X(1.0025)^{-24} = 25000((1.0025)^{-108} + (1.0025)^{-180}) = 35040.58$, which gives $X = (35040.58 - 20000)(1.0025)^{24} = \$15,969.44$.

7. *Mr. Fowler wants to save up \$1,400,000 for his retirement in 24 years time.*

   (a) If his effective annual rate of interest is 5%, how much does he need to invest now?

   $1400000(1.05)^{-24} = \$434,095.07$

   (b) Suppose he invests this much, but the actual rate of interest he receives is 4.5%. How much longer does he have to wait before he can retire with this much money?

   $434095.07(1.045)^n = 1400000$, so $1.045^n = 3.2251$, so $n = \frac{\log(3.2251)}{\log(1.045)} = 26.60$, so he has to wait an additional 2.6 years to retire.

8. *What rate of simple discount is equivalent to 7% simple interest over a period of 9 months?*

   $1 - d\frac{9}{12} = \left(1 + 0.07\frac{9}{12}\right)^{-1}$, so $d = \frac{12}{9}\left(1 - \left(1 + 0.07\frac{9}{12}\right)^{-1}\right) = 6.65\%$.

9. *What price should you pay for a T-bill of face value \$8,000, maturing in 62 days at a simple interest rate of 5%?*

   $P\left(1 + 0.05\frac{62}{365}\right) = 8000$, so $P = \frac{8000}{1+0.05\frac{62}{365}} = 7932.63$.

10. *Which of the following interest rates is best for the investor?*

    (i) 6.8% compounded quarterly

    The annual effective rate is $(1.017)^4 - 1 = 6.98\%$.

    (ii) 6.9% compounded annually

    (iii) 6.7% compounded daily

    The annual effective rate is $\left(1 + \frac{0.067}{365}\right)^{365} - 1 = 6.93\%$. Therefore, (i) is best for the investor, since it gives the highest annual effective rate.

11. *What rate of compound interest is equivalent to a 6% rate of compound discount?*

    We have $i = 0.94^{-1} - 1 = 6.38\%$.

12. *Mr. Grant wants to buy a car. Today, he would need to pay $52,000 to buy the car, but he only has $34,000. He invests this money at $j_{12} = 4\%$. However, the downpayment needed increases with inflation at a rate of $j_{12} = 2\%$. How long must he wait before he has saved up enough to buy the car?*

    We need to solve $34000(1.003333333)^n = 52000(1.001666667)^n$, or $\left(\frac{1.003333333333}{1.00166666666667}\right)^n = \frac{52000}{34000}$, so $n = \frac{\log(1.5294)}{\log(1.001664)} = 255.57$ months, so he needs to wait 256 months or 21 years 4 months.

13. *What rate of continuous compounding (constant force of interest) is equivalent to annual effective rate of 2%?*

    We need $e^\delta = 1.02$, so $\delta = \log(1.02) = 0.0198$.

14. *If force of interest is given by $\delta_t = 0.05 + 0.1t - 0.06t^2$ over a two-year period, how much needs to be invested at the start of that period to have $150,000 by the end of the period.*

    We have $Xe^{\int_0^2 (0.05+0.1t-0.06t^2)dt} = 150000$. We have $\int_0^2 (0.05 + 0.1t - 0.06t^2)dt = [0.05t + 0.05t^2 - 0.02t^3]_0^2 = 0.1 + 0.2 - 0.16 = 0.14$, so we have $X = 150000e^{-0.14} = \$130,403.74$.