MATH 3030, Abstract Algebra
FALL 2012
Toby Kenney

Homework Sheet 8

Due: Friday 23rd November: 3:30 PM

Basic Questions

1. Find the remainder of 6^{12345} when divided by 13.

2. Find the remainder when 9^{123456} is divided by 91. [Hint: $91 = 7 \times 13$; see Q. 7.]

3. Find the last digit of $3^{3^{3^{3^{3^{3^{3^{3^{3^{3^{3^{3}}}}}}}}}}}$ (in base 10).

4. Solve:

   (a) $15x \equiv 11(mod 33)$

   (b) $5x \equiv 11(mod 33)$

5. Describe the field of quotients of the integral domain $\{a + b\sqrt{2}i | a, b \in \mathbb{Z}\}$.

6. Describe the field of quotients of the integral domain $\{a + b\sqrt{5} | a, b \in \mathbb{Z}\}$.

Theoretical Questions

7. Let $n = pq$ where p and q are prime.

   (a) Show that $\phi(n) = (p - 1)(q - 1)$.

   (b) If e and $n = pq$ are known numbers, and we are told m^e modulo n, how can we recover the value of m?

8. Prove Wilson's Theorem, that if p is prime, then $(p - 1)! \equiv -1(\mod p)$. [Hint: first show that 1 and -1 are the only self-inverse elements of $\mathbb{Z}_p$.]

9. Prove the distributive law holds in the field of quotients of an integral domain.

10. If D' is a subdomain of D, must the field of quotients of D' be a subfield of the field of quotients of D?