MATH/STAT 3360, Probability
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In Class Examples

Basic Principal of Counting

Question

A statistics textbook has 8 chapters. Each chapter has 50 questions. How many questions are there in total in the book?

Question

How many 9-digit numbers are there that contain each of the digits 1–9 once.

Permutations

Question

A band are planning a tour of Canada. They are planning to perform in 15 venues across Canada, before returning to their home in Halifax. They want to find the cheapest way to arrange their tour. The only way to do this is to work out the cost of all possible orderings of the venues. They have a computer, which can calculate the cost of 100,000,000 orderings of venues every second.
(a) How long will it take the computer to calculate the cost of all possible orderings of the venues?
(b) What if they want to extend the tour to 18 venues?

Permutations

Question

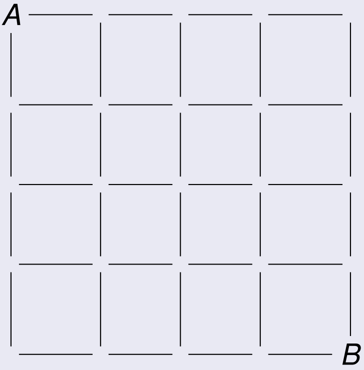
In an Olympic race with 8 contestants, how many possibilities are there for the medal finishes? That is, how many different ways are there to choose 3 contestants in order?

Combinations

Question

How many 3-element subsets are there in a 12-element set?

Question



In the above diagram, how many shortest paths are there from A to B?

Multinomial Coefficients

Question

How many distinct ways can the letters of the word “PROBABILITY” be arranged.

Multinomial Coefficients

Question

A professor decides to award a fixed number of each grade to his students. He decides that among his class of 10 students, he will award one A, two B, three C, three D and one fail. How many different results are possible in the course.

Multinomial Coefficients

Question

In the polynomial $(x + y + z)^6$, what is the coefficient of $x^2 y^3 z$?

Question

How many ways are there to divide a class of 20 students into 10 pairs if the order of the 10 pairs doesn't matter?

Sample Spaces and Events

Question

For events A, B, C and D, how are the events $(AB) \cup (CD)$ and $(A \cup C)(B \cup D)$ related?

Question

For an experiment with sample space $\{1, 2, 3, 4, 5\}$, is there a probability P such that $P(\{1, 2\}) = 0.4$, $P(\{1, 3\}) = 0.6$ and $P(\{2, 3\}) = 0.1$?

Axioms of Probability

Question

For an experiment with sample space $\{1, 2, 3, 4, 5\}$, is there a probability P such that $P(\{1, 2, 3\}) = 0.4$, $P(\{1, 3, 4\}) = 0.6$ and $P(\{2, 4, 5\}) = 0.1$?

Question

For events A, B and C, we know that $P(A) = 0.6$, $P(B) = 0.5$ and $P(C) = 0.4$, and that $P(AB) = 0.4$, $P(AC) = 0.3$ and $P(BC) = 0.2$, and $P(ABC) = 0.1$. What is $P(A \cup B \cup C)$.

Question

For events A, B and C, we know that $P(A) = 0.5$, $P(B) = 0.7$ and $P(C) = 0.3$, and that $P(AB) = 0.3$, $P(AC) = 0.2$ and $P(BC) = 0.2$, and $P(A \cup B \cup C) = 0.9$. What is $P(ABC)$?

Simple Propositions

Question

A fair coin is tossed 5 times.
(a) What is the probability that the sequence HHH occurs somewhere in the 5 tosses?
(b) What is the probability that the sequence THT occurs somewhere in the 5 tosses?

Question

What is the probability that a number chosen uniformly at random in the range 1–1,000,000 is divisible by at least one of 2, 3, or 5?

Question

You are planning to learn some languages. You want to maximise the number of people with whom you will be able to speak, or equivalently, the probability that you will be able to speak with a randomly chosen person. You perform some preliminary research and find out the following:

Language	Proportion of people who speak it
Chinese	19.4%
English	7.3%
Hindi	7.1%
Spanish	6.0%
Arabic	4.0%

...

Simple Propositions

Question (continued)

You also investigate what proportion of people speak each pair of languages, as summarised in the following table.

	Chinese	English	Hindi	Spanish	Arabic
Chinese		0.2%	0.1%	0.0%	0.1%
English	0.2%		1.8%	1.6%	0.7%
Hindi	0.1%	1.8%		0.0%	0.0%
Spanish	0.0%	1.6%	0.0%		0.1%
Arabic	0.1%	0.7%	0.0%	0.1%	

If you assume that the number of people speaking three languages is small enough to be neglected, which three languages should you learn in order to maximise the number of people to whom you can speak?

Question

What is the probability that 13 randomly chosen cards from a standard deck include all four aces?

Question

A particular genetic disease affects people if and only if they have two defective copies of the gene in question. [Everyone has two copies of the gene, one from each parent, and passes on one copy at random to each of their children.] If both members of a couple have a sibling with the disease (but no parents with the disease)
(a) what is the probability that their first child will have the disease?
(b) If they have two children, what is the probability that both will have the disease?

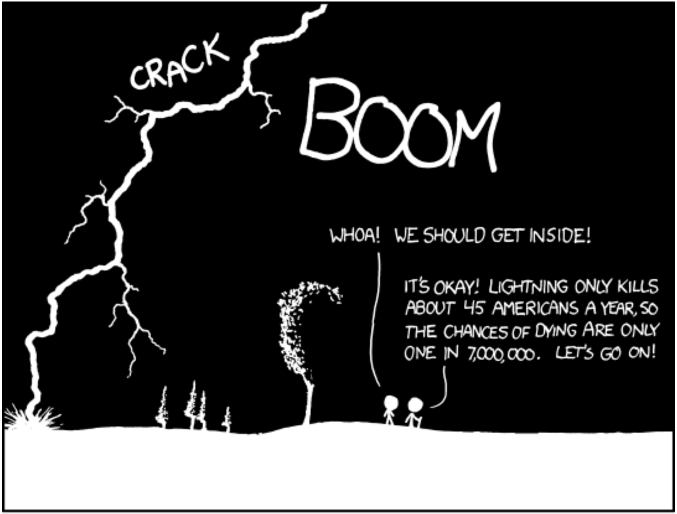
Sample Spaces of Equally Likely Events

Question

What is the probability that a poker hand (with five cards in total) contains exactly three Aces?

Question

What is the probability that the sum of two dice is at least 7 conditional on it being odd?



THE ANNUAL DEATH RATE AMONG PEOPLE WHO KNOW THAT STATISTIC IS ONE IN SIX.

Conditional Probability

Question

Two of your friends often talk to you about the books they like. From experience, you have found that if Andrew likes a book, the probability that you will also like it is 70%, while if Brian likes a book, the probability that you will also like it is 40%. You are shopping with another fried, who tells you that she remembers Andrew telling her that he liked the book you are considering buying, but she's not completely sure that it was Andrew who told her, rather than Brian. If the probability that it was Andrew who told her is 80%, what is the probability that you will like the book?

Conditional Probability

Question

Two fair 6-sided dice are rolled.
(a) What is the probability that the larger of the two numbers obtained is 5, given that the smaller is 3. [By "the smaller number is 3" I mean that one of the numbers is 3, and the other is at least 3, so that if the roll is (3,3), the smaller number is 3.]
(b) One of the dice is red, and the other is blue. What is the probability that the number rolled on the red die is 5 given that the number rolled on the blue die is 3 and the number rolled on the red die is at least 3?
(c) Why are the answers to (a) and (b) different?

Conditional Probability

Question

You are a contestant in a game-show. There are three doors, numbered 1, 2, and 3, one of which has a prize behind it. You are to choose a door, then the host will open another door to show that the prize is not behind it (if the door you chose has the prize behind it, he chooses which door to open at random). You then have the opportunity to switch to the other unopened door.
(a) Should you switch to door 3?
(b) What is your probability of winning (assuming you make the best choice of whether or not to switch)?

1
2
3
G
i'm gonna pick door number 2
excellent choice there jason! just for fun, i'm going to show you what is behind door number 1... this is what you could have won if you picked door number 1
DOOR 1
you could have won a banana!
only doors 2 and 3 are left... that means...
$1,000,000
is behind one of these doors...
now i'm only going to ask you once so listen carefully...
do you want to switch to door 3?
ABSOLUTELY
okay jason, let's open door number 3 to see what you have won...
spikedmath.com
© 2009
DOOR 3
you won a banana
BOOOOOOOOO....
why the heck would he switch... NEVER switch!!
what a moron!!
BOOO...
damn you statistics!!
damn you to hell !!!

Question

You are a contestant in a game-show. There are three doors, numbered 1, 2, and 3, one of which has a prize behind it. You are to choose a door, then the host will open another door to show that the prize is not behind it (if the door you chose has the prize behind it, he chooses which door to open at random). You then have the opportunity to switch to the other unopened door. In this case, you have cheated and know that the prize is not behind door 2. You choose door 1, and the host opens door 2.
(a) Should you switch to door 3?
(b) What is your probability of winning (assuming you make the best choice of whether or not to switch)?

Bayes' Theorem

Question

A test for a particular disease has probability 0.005 of giving a positive result if the patient does not have the disease (and always gives a positive result if the patient has the disease). The disease is rare, and the probability that an individual has the disease is 0.0001.
(a) An individual is tested at random, and the test gives a positive result. What is the probability that the individual actually has the disease?
(b) A patient goes to the doctor with a rash, which is a symptom of this rare disease, but can also be caused by other problems. The probability that a person without the disease has this rash is 0.01. If the patients test comes back positive, what is the probability that he has the disease?

Question

A civil servant wants to conduct a survey on drug abuse. She is concerned that people may not admit to using drugs when asked. She therefore asks the participants to toss a coin (so that she cannot see the result) and if the toss is heads, to answer the question "YES", regardless of the true answer. If the toss is tails, they should give the true answer. That way, if they answer "YES" to the question, she will not know whether it is a true answer. Suppose that 5% of the survey participants actually use drugs. What is the probability that an individual uses drugs, given that they answer "YES" to the question on the survey?

Independent Events

Question

4 fair coins are tossed. Which of the following pairs of events are independent?

(a)	The first coin is a head.	There are exactly two heads among the four tosses.
(b)	There are exactly two heads among the first three tosses	There are exactly two heads among the last three tosses.
(c)	The sequence `TTH` occurs in the 4 tosses.	The sequence `THT` occurs.

Independent Events

Question

Three cards are drawn from a standard deck. Which of the following pairs of events are independent?

(a)	The first card is a heart.	There are exactly two queens among the three cards.
(b)	At least two of the cards are hearts	At least two of the cards are 9s.
(c)	All three cards are red	None of the cards is an Ace

Conditional Probability is a Probability

Question

For three events A, B and C, we know that $P(A|C) = 0.4$, $P(B|C) = 0.3$, $P(ABC) = 0.1$, $P(C) = 0.8$. Find $P(A \cup B|C)$.

Expected Value

Question

What is the expected value of the product of the numbers rolled on two fair 6-sided dice.

Expectation of a Function of a Random Variable

Question

You are bidding for an object in an auction. You do not know the other bids. You believe that the highest other bid (in dollars) will be unifomly distributed in the interval $(100, 150)$. You can sell the item for \$135. What should you bid in order to maximise expected profit?

Variance

Question

Two fair 6-sided dice are rolled, and the sum is taken. What is the variance of this sum?

Question

An insurance company sets it's premium for car insurance at \$500. They estimate that the probability of a customer making a claim is $\frac{1}{200}$, in which case they will pay out \$5,000. They sell policies to 100 customers. After costs, the premiums are enough for them to pay out up to 5 claims. What is the probability that the insurance company will have to pay out more than 5 claims?

Question

An insurance company insures 20,000 homes. The probability of a particular home making a claim for $1,000,000 is $\frac{1}{15{,}000}$.
(a) What is the probability that the company receives more than 2 claims? [Calculate the exact probability.]
(b) What probability do we calculate for (a) if we use the Poisson approximation?

Poisson Random Variables

Question

Suppose the number of cars that want to park on a particular street each day is a Poisson random variable with parameter 5. There are 4 parking spots on the street.
(a) What is the probability that on a given day, none of the parking spots is used?
(b) What is the probability that one of the cars wanting to park on that street will not be able to?
(c) What is the expected number of parking spots unused on that street?

Expectation of a Function of a Random Variable

Question

The number of customers a company has on a given day is a Poisson random variable with parameter 5. For each customer, the company receives \$300, but to handle n customers, the company has to pay $100n + 500 + \frac{600}{n+1}$. What is the companies expected profit each day?

Expectation of Sums of Random Variables

Question

100 fair 6-sided dice are rolled, what is the expected value of the total of all 100 dice.

Question

A gambler starts with \$5, and continues to play a game where he either wins or loses his stake, with equal probability, until he either has \$25 or has \$0. He is allowed to choose how much to bet each time, but is not allowed to bet more than he has, and does not bet more than he would need to win to increase his total to \$25. He decides how much to bet each time by rolling a fair (6-sided) die, and betting the minimum of the number shown, the amount he has, and the amount he needs to increase his total to \$25. What is the probability that he quits with \$25? [You may assume that he will certainly reach either \$25 or \$0 eventually. Hint: some of the information in this question is not necessary.]

Question

You are considering an investment. You would originally invest \$1,000, and every year, the investment will either increase by 60% or decrease by 60%, with equal probability. You plan to use the investment after 20 years.

(a) What is the expected value of the investment after 20 years?

(b) What is the probability that the investment will be worth more than \$1,000 after 20 years?

Question

What is the cumulative distribution function for the sum of two fair dice?

Expectation and Variance of Continuous Random Variables

Question

A random variable X has probability density function
$f_X(x) = \frac{2}{\sqrt{3}\pi(1+\frac{x^2}{3})^2}$.

(a) What is the expectation $\mathbb{E}(X)$?

(b) What is the variance $\mathrm{Var}(X)$? [Hint: differentiate $\frac{1}{1+\frac{x^2}{3}}$, and use this to integrate by parts. Recall that $\frac{d \arctan\left(\frac{x}{\sqrt{3}}\right)}{dx} = \frac{1}{\sqrt{3}\left(1+\frac{x^2}{3}\right)}$, and that $\arctan(-\infty) = -\frac{\pi}{2}$ and $\arctan(\infty) = \frac{\pi}{2}$.]

Expectation and Variance of Continuous Random Variables

Question

A random variable X has a truncated exponential distribution — that is, its probability density function is given by:

$$f_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{\lambda e^{-\lambda x}}{1-e^{-\lambda a}} & \text{if } 0 \leqslant x < a \\ 0 & \text{if } a \leqslant x \end{cases}$$

(a) What is the expectation $\mathbb{E}(X)$?
(b) What is the variance $\mathrm{Var}(X)$?

Question

Which of the following distributions for X gives the larger probability that $X > 3$?
(i) X is uniformly distributed on $(-2, 7)$.
(ii) X is uniformly distributed on $(0, 5)$?

Question

Let X be normally distributed with mean 1 and standard deviation 1. What is the probability that $X^2 > 1$?

Question

Let X be normally distributed with mean 4 and standard deviation 3. What is the shortest interval such that X has an 80% probability of lying in that interval?

Question

An online banking website asks for its customers date of birth as a security question. Assuming that the age of customers who have an online banking account is normally distributed with mean 35 years and standard deviation 10 years:

(a) how many guesses would a criminal need to make in order to have a 50% chance of correctly guessing the date of birth of a particular customer?

(b) If the criminal estimates that he can safely make up to 200 guesses without being caught, what is the probability of guessing correctly.

Question

A car insurance company sets its premium at \$450 for a year. Each customer has a probability $\frac{1}{100}$ of making a claim for \$10,000. The company has to cover adminstrative costs of \$300 per customer. How many customers does it need in order to have a probability 0.001 of being unable to cover its costs? [You may use any reasonable approximations for the distribution of the number of claims.]

Normal Random Variables

Question

The number of visitors to a particular website at a given time is normally distributed with mean 1,300 and variance 500^2. The server that hosts the website can handle 2,500 visitors at once.

(a) What is the probability that there are more visitors than it can handle?

(b) The company wants to reduce the chance of having too many visitors to 0.0002. How many visitors does the server need to be able to handle in order for this to be achieved?

(c) The company also wants to attract more visitors to the website, so it upgrades the server so that it can handle 3,500 visitors at once. Assuming that the variance remains at 500^2, what is the largest mean number of visitors at a given time, such that the company can maintain its 0.0002 target for the probability of having too many visitors?

Question

A scientist, Dr. Jones believes he has found a cure for aging. If he is right, people will no longer die of old age (but will still die of disease and accidents at the same rate). In this case, what would life expectancy be for people who do not grow old? [You may assume that people currently die from accidents and disease at a roughly constant rate before the age of about 50, and that after Dr. Jones' cure, they will continue die at this rate forever. Currently 93% of people live to age 50.]

Question

If X is an exponential random variable with parameter λ, what is the distribution function of X^2?

Distribution of a Function of a Random Variable

Question

Calculate the probability density function of the square of a normal random variable with mean 0 and standard deviation 1.

Question

A fair (6-sided) die is rolled 50 times. What is the probability of getting exactly 7 sixes and 6 fives?

Question

A fair (6-sided) die is rolled 10 times. What is the probability of getting the same number of '5's and '6's?

Independent Random Variables

Question

Give an example of a joint distribution function for two continuous random variables X and Y such that for any x for which $f_X(x) > 0$, $\mathbb{E}(Y|X = x) = 0$ and for any y for which $f_Y(y) > 0$, $\mathbb{E}(X|Y = y) = 0$, but such that X and Y are not independent.

Joint Probability Distribution of Functions of Random Variables

Question

(R, Θ) have joint density function $f_{(R,\Theta)}(r, \theta) = \frac{1}{2\pi} re^{-\frac{r^2}{2}}$. Let $X = R\cos\Theta$ and $Y = R\sin\Theta$.

(a) What is the joint density function of (X, Y)?

(b) Are X and Y independent?

Question

Show that the minimum of two independent exponential random variables with parameters λ_1 and λ_2 is another exponential random variable with parameter $\lambda_1 + \lambda_2$.

Question

If X is uniformly distributed on $(0, 2)$ and Y is uniformly distributed on $(-1, 3)$, what is the probability density function of $X + Y$?

Question

The number of claims paid out by an insurance company in a year is a Poisson distribution with parameter λ. The number of claims it can afford to pay is n. It is considering merging with another insurance company which can afford to pay m claims and which will have to pay out Y claims, where Y is a poisson distribution with parameter μ. Under what conditions on m,n,λ and μ will the considered merger reduce the company's risk of bankrupcy?[You may use the normal approximation to the Poisson for estimating the risk of bankrupcy.]

Question

X is normally distributed with mean 2 and standard deviation 1. Y is independent of X and normally distributed with mean 5 and standard deviation 3.
(a) What is the distribution of $X + Y$?
(b) What is the probability that $Y - X > 1$?

Conditional Distributions: Discrete Case

Question

(a) What is the joint distribution function of the sum of two fair dice, and the (unsigned) difference [e.g., for the roll $(3, 6)$ or $(6, 3)$ the difference is 3]?
(b) What is the conditional distribution of the difference, conditional on the sum being 7?

Conditional Distributions: Continuous Case

Question

X is a normal random variable with mean 0 and variance 1. Y is given as $Y = X^3 + N(0, 2^2)$. What is the conditional distribution of X given $Y = 2$? [You do not need to calculate the normalisation constant.]

Conditional Distributions: Continuous Case

Question

If the point (X, Y) is uniformly distributed inside the set $\{(x, y)|x^2 + 2y^2 - xy < 5\}$, what is the conditional distribution of X given that $y = 1$.

Question

A company is planning to run an advertising campaign. It estimates that the number of customers it gains from the advertising campain will be approximately normally distributed with mean 2,000 and standard deviation 500. It also estimates that if it attracts n customers, the time until the advertising campaign becomes profitable is exponentially distributed with parameter $\frac{n}{500}$.
(a) What is the joint density function for the number of new customers and the time before this advertising campaign becomes profitable?
(b) What is the marginal distribution of the time before this campaign becomes profitable?

Question

Consider the following experiment: Roll a fair (6-sided) die. If the result n is odd, toss n fair coins and count the number of heads. If n is even, roll n fair dice and take the sum of the numbers. What is the expected outcome?

Expectation of Sums of Random Variables

Question

A company has 50 employees, and handles a number of projects. The time a project takes (in days) follows an exponential random variable with parameter $\frac{k}{100}$, where k is the number of employees working on that project. The company receives m projects simultaneously, and has no other projects at that time. What is the expected time until completion of a randomly chosen project if:
(a) The company divides its workers equally between the m projects, and starts all projects at once? [Ignore any requirements that the number of employees working on a given project should be an integer.]
(b) The company orders the projects at random, and assigns all its workers to project 1, then when project 1 is finished, assigns all its workers to project 2, and so on?

Question

A government is trying to demonstrate that speed cameras prevent accidents. It conducts the following study:

(i) Pick 10 locations.

(ii) Count the number of accidents at each location in the past 3 years.

(iii) Choose the location x with the most accidents, and install a speed camera there.

(iv) Count the number of accidents at location x in the following 3 years.

(v) The number of accidents "prevented" is the number of accidents at location x during the 3 years before installing the speed camera, minus the number of accidents at location x in the 3 years after installing the speed camera.

Expectation of Sums of Random Variables

...

Assume that the number of accidents at each site is an independent Poisson random variable with parameter 0.5, and that installing a speed camera at a given location has no effect on the number of accidents at that location. What is the expected number of accidents "prevented". [You may ignore the possibility that more than 4 accidents happen at any location, to get a good approximation to the true answer. You may wish to use the expectation formula $E(X) = \sum_{n=1}^{\infty} P(X \geqslant n)$ for a random variable X taking values in the natural numbers.]

Question

In a particular area in a city, the roads follow a rectangular grid pattern. You want to get from a location A to a location B that is m blocks East and n blocks North of A.

(a) How many shortest paths are there from A to B?

(b) After a storm, each road segment independently has probability p of being flooded. What is the expected number of shortest paths from A to B that are still useable?

(d) [Chapter 8] Use Markov's inequality to find a bound on the probability that there is still a useable shortest path from A to B.

Question

A random graph is generated on n vertices where each edge has probability p of being in the graph. What is
(a) The expected number of triangles?
(b) The variance of the number of triangles? [Hint: there are two types of pairs of triangles to consider — those that share an edge, and those that don't.]

Question

An ecologist is collecting butterflies. She collects a total of n butterflies. There are a total of N species of butterfly, and each butterfly is equally likely to be any of the species.

(a) What is the expected number of species of butterflies she collects?

(b) What is the variance of the number of species of butterflies she collects?

Question

Show that $\mathrm{Cov}(X, Y) \leqslant \sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}$. [Hint: consider $\mathrm{Var}(\lambda X - Y)$ for suitable λ.]

Conditional Expectation

Question

You are considering an investment. The initial investment is \$1,000, and every year, the investment increases by 50% with probability 0.4, decreases by 10% with probability 0.3, or decreases by 40% with probability 0.3. Let X_n denote the value of the investment after n years [so $X_0 = 1000$].
(a) Calculate $\mathbb{E}(X_n | X_{n-1} = x)$.
(b) Calculate $\mathbb{E}(X_{10})$.
(c) What is the probability that the investment is actually worth the amount you calculated in part (b), or more?

Conditional Expectation

Question

An outbreak of a contagious disease starts with a single infected person. Each infected person i infects N_i other people, where each N_i is a random variable with expectation $\mu < 1$. If the N_i are all independent and identically distributed, what is the expectation of the total number of people infected?

Question

If X is exponentially distributed with parameter λ and Y is uniformly distributed on the interval $[a, b]$, what is the moment generating function of $X + 2Y$?

Moment Generating Functions

Question

If X is exponentially distributed with parameter λ_1 and Y is exponentially distributed with parameter λ_2, where $\lambda_2 > \lambda_1$:
(a) find the moment generating function of $X - Y$.
(b) [Chapter 8] Use the Chernoff bound with $t = \frac{\lambda_1 - \lambda_2}{2}$ to obtain a *lower* bound on the probability that $X > Y$.

Question

Let X, Y have a multivariate normal distribution, such that X has mean 2 and standard deviation 1; Y has mean 4 and standard deviation 2; and $\mathrm{Cov}(X, Y) = 2$. What is $P(Y > X)$?

Question

Use Markov's inequality to obtain a bound on the probability that the product of two independent exponentially distributed random variables with parameters λ and μ is at least 1.

Question (From Section 7)

In a particular area in a city, the roads follow a rectangular grid pattern. You want to get from a location A to a location B that is m blocks East and n blocks North of A.

(d) Use Markov's inequality to find a bound on the probability that there is still a useable shortest path from A to B.

Markov's Inequality, Chebyshev's Inequality and the Weak Law of Large Numbers

Question

X is uniformly distributed on $[3, 8]$. Y is a positive random variable with expectation 3. X and Y are independent. Obtain an upper bound on the probability that $XY > 20$.

(a) By using Markov's inequality on XY.

(b) By conditioning on $X = x$ and using Markov's inequality on Y to bound $P\left(Y \leqslant \frac{20}{x}\right)$.

(c) [bonus] Without any assumptions on the independence of X and Y, find an upper bound on $P(XY \geqslant 20)$. [Condition on the value of Y, and consider the conditional expectation $\mathbb{E}(X|Y)$.]

Central Limit Theorem

Question

A car insurance company is trying to decide what premium to charge. It estimates that it can attract 20,000 customers. Each customer has a probability $\frac{1}{100}$ of making a claim for \$3,000. The company has to cover \$1,000,000 in costs. What premium should it set in order to have a probability 0.00001 of being unable to cover its costs?

Central Limit Theorem

Question

An insurance company insures houses. They find that the expected value of the total amount claimed by an insured household each year is \$500, and the standard deviation is \$3,000. The company charges an annual premium of \$600 for each household. Of this premium, \$60 is needed to cover the company's costs. (So the remaining \$540 can be used to cover claims.)

(a) If the company insures 50,000 homes, what is the probability that it makes a loss in a given year? (That is, the probability that it must pay out more in claims and costs than it collects in premiums.)

(b) What important assumptions have you made in part (a)? How reasonable are these assumptions?

(c) If the company wants to reduce this probability to less than 0.0002, how many houses would it need to insure at the current premium?

Question

A gambler is playing roulette in a casino. The gambler continues to bet \$1 on the number where the ball will land. There are 37 numbers. If the gambler guesses correctly, the casino pays him \$35 (and he keeps his \$1). Otherwise, he loses his \$1. If he keeps playing, how long is it before the probability that he has more money than he started with is less than 0.0001?

Central Limit Theorem

Question

A civil servant is conducting a survey to determine whether people support a proposed policy. They plan to survey 200 people. Suppose that in fact 48% of people support the proposed policy, and that the people surveyed are chosen at random, so that each person surveyed independently supports the policy with probability 0.48.
(a) What is the probability that the survey leads them to wrongly conclude that a majority of people support the policy, i.e. at least 101 people in the survey support the policy. [You may use any reasonable approximations for the distribution of the number of people in the survey that support the policy.]
(b) How many people would they need to survey to have a 99% chance of reaching the correct conclusion (which happens if strictly less than half of the people in the survey support the policy).

Question (Section 7)

If X is exponentially distributed with parameter λ_1 and Y is exponentially distributed with parameter λ_2, where $\lambda_2 > \lambda_1$:
(a) find the moment generating function of $X - Y$.

Answer

The moment generating function is:

$$M_{X-Y}(t) = \frac{\lambda_1 \lambda_2}{(\lambda_1 - t)(\lambda_2 + t)}$$

Question (continued)

(b) [Chapter 8] Use the Chernoff bound with $t = \frac{\lambda_1 - \lambda_2}{2}$ to obtain a *lower* bound on the probability that $X > Y$.

Question

You are investing your money in the stock market. In year i, the value of your investment increases by $X_i\%$ where the X_i are i.i.d. random variables with mean 2. That is, if the value of your investment at the start of year i is I_i, then the value at the end of year i is given by $I_{i+1} = I_i \left(1 + \frac{X_i}{100}\right)$. After n years, your *rate of return* is given by $r_n = \left(\left(\frac{I_n}{I_0}\right)^{\frac{1}{n}} - 1\right) \times 100\%$ where I_n is the value of your investment after n years (so I_0 is your initial investment). [This r gives the annual percentage interest that would result in a total I_n after n years, starting from I_0.]

(a) What can you say about the behaviour of r_n as $n \to \infty$? [Hint: think about $Y_i = \log\left(1 + \frac{X_i}{100}\right)$, and $J_n = \log\left(\frac{I_n}{I_0}\right)$.]

. . .

Question (continued)

(b) A friend tells you that if the amount you have after i years is I_i, then the expected amount that you have after $i + 1$ years is $1.02 \times I_i$. Therefore, since the amount by which the value of your investment changes each year is independent, after n years, the expected value of your investment will be $(1.02)^n \times I_0$. Explain the relationship between this equation and your answer to (a).