ACSC/STAT 3703, Actuarial Models I (Further Probability with Applications to Actuarial Science)

Winter 2015

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Homework Sheet 1

Due: Friday 23rd January: 12:30 PM

Basic Questions

1. The survival function for an inverse Weibull distribution is given by $S(x) = 1 - e^{\left(\frac{\theta}{x}\right)^\tau}$. Calculate the hazard-rate.

2. A continuous random variable has moment generating function given by $M(t) = (1-4t)^{-2}e^{1-\sqrt{1-2t}}$ for $t < \frac{1}{2}$. Calculate its coefficient of variation.

3. Calculate the mean excess loss function for a distribution with survival function given by $S(x) = \left(1 - \frac{x}{130}\right)^{\frac{1}{5}}$.

4. Find the equilibrium distribution for a Weibull distribution with survival function given by $S(x) = e^{-\left(\frac{x}{\theta}\right)^\tau}$.

Standard Questions

5. A Burr distribution has survival function

$$S(x) = \left(\frac{1}{1+\left(\frac{x}{\theta}\right)^\gamma}\right)^\alpha$$

Consider the two Burr distributions $\alpha = 2, \gamma = 3, \theta = 20$ and $\alpha = 3, \gamma = 2, \theta = 40$. Which has the heavier tail when measured by the hazard rate function?

6. An insurance company is trying to fit a paralogistic distribution to its claims data. The survival function for this distribution is given by

$$S(x) = \left(\frac{1}{1+\left(\frac{x}{\theta}\right)^\alpha}\right)^\alpha$$

It is very important for the insurance company to correctly model the expected value and the 95th percentile of this distribution. The company therefore chooses α and θ so that these values match their observed mean of 2,300 and their observed 95th percentile of 6,700. Which of the following

values should they choose for α, and what should be the corresponding value of θ?

(i) 1.2131

(ii) 1.3871

(iii) 1.8786

(iv) 2.4322