ACSC/STAT 3720, Life Contingencies I
Winter 2015

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Homework Sheet 1

Due: Friday 23rd January: 10:30 PM

Basic Questions

1. An insurance company models the future lifetime of an individual as having survival function $S(x) = e^{-\left(\frac{x}{80}\right)^2}$. Calculate

   (a) Force of mortality.

   (b) The mean and standard deviation of T_x. [Hint: to integrate functions of the form e^{-ax^2}, think about the probability density function of a normal distribution.]

   (c) The mean curtate future lifetime. [You do not need to sum the series, just write the correct formula for the sum.]

2. An insurance company uses a survival model with survival function ${}_tp_x = \left(1 - \frac{t}{\omega - x}\right)^{\frac{1}{8}}$, where ω is the maximum age attainable. The company wants to ensure that the life expectancy of an individual aged 65 under this model is 16 years. What age should they choose as the maximum age attainable?

3. An insurance company uses a survival model given by

$$S_0(x) = \frac{2}{3}\left(1 - \frac{x}{100}\right)^{\frac{1}{6}} + \frac{1}{3}\left(1 - \frac{x}{125}\right)^{\frac{1}{4}}$$

   Using this model, prepare a life table for the ages from 35 to 40, using radix 10,000.

4. Using the lifetable:

x	l_x	d_x
65	10000.00	30.89
66	9969.11	33.45
67	9935.66	36.23
68	9899.43	39.22
69	9860.21	42.46
70	9817.75	45.95
71	9771.80	49.71
72	9722.09	53.76

   calculate the probability that an individual aged 65 years and two months survives another 5 years, using:

   (a) the uniform distribution of deaths assumption.

   (b) the constant force of mortality assumption.

Standard Questions

5. An insurance company wants to use a model of mortality of the form $\mu_x = \frac{a}{b-x}$. Based on the company's data, the life expectancy for an individual aged 65 is 19 years, and the probability of an individual aged 44 surviving for 10 years is 0.982. It is extremely important for these properties to match the observed data. The company chooses the values of a and b to ensure that these observations are matched by the model. Which of the following is the correct value of a? What is the corresponding value of b? Justify your answer.

   (i) 0.0443

   (ii) 0.0654

   (iii) 0.102

   (iv) 0.288

6. An insurance company prepares the following lifetable for an unhealthy individual.

x	l_x	d_x
45	10000.00	602.13
46	9397.87	683.49
47	8714.37	765.53
48	7948.85	843.45
49	7105.40	910.70
50	6194.70	959.07
51	5235.63	979.14
52	4256.49	961.56
53	3294.93	899.13
54	2395.80	789.74
55	1606.07	639.52

   After a medical examination, it determines that the individual's probability of death at age 45 is only 0.0023. The probability of death in each subsequent year remains the same. Prepare a new life table for this individual over the same range using radix 10,000.