# ACSC/STAT 3720, Life Contingencies I <br> Winter 2015 <br> Toby Kenney <br> Due: Monday 23rd February: 10:30 PM 

## Basic Questions

1. Using the lifetable in Table 1, calculate $\ddot{a}_{[35]+3}$ at interest rate $i=0.06$.
2. An individual aged 49 for whom Table 1 is appropriate, takes out a 10 -year term-insurance policy. The annual premiums are $\$ 7,000$, payable at the begining of each year. If the current interest rate is $i=0.05$, what is the expected present value of the premiums received?
3. An annuity pays out continuously at a rate of $\$ 3,000$ a year until the death of a select individual currently aged 67 to whom the lifetable in Table 1 applies. What is the expected present value of this annuity, using the uniform distribution of deaths assumption, and force of interest $\delta=0.09$ ?
4. A pension plan pays monthly benefits of $\$ 2500$ to an individual aged 68 . What is the expected present value of the benefit under the uniform distribution of deaths assumption, interest rate $i^{(12)}=0.06$ and the lifetable in Table 1? [These allow us to calculate $A_{68}=0.240986$.]

## Standard Questions

5. A pension plan pays an annual benefit of $\$ 17,000$ to an individual aged 69 , for whom the ultimate part of the lifetable in Table 1 applies. The interest rate is $i=0.08$, which gives $A_{69}=0.18358$ and $A_{79}=0.300473$. The individual wants to change the policy to have guaranteed payments for the first 10 years, but keep the EPV of the benefits the same. What should the new annual payments be?
6. A man aged 66 , to whom the ultimate part of the lifetable in Table 1 applies, wants a pension which will pay $\$ 20,000$ in a year's time, and thereafter will provide annual pensions increasing by $4 \%$ every year (so the second payment when the man turns 68 will be $\$ 20,800$ ). What is the expected present value of the benefits of this pension if the current interest rate is $i=0.11$ ?
7. A woman aged 46 is receiving a pension of $\$ 27,000$ at the start of each year. She wants to change this to a monthly pension. If the appropriate life table is in Table 1 and the interest rate is $i=0.04$, then we can calculate $A_{46}=0.178312$. Use Woolhouse's formula to calculate the monthly pension that has the same expected present value.
8. An individual aged 48 is starting to invest in a pension plan. He wants to receive $\$ 26,000$ a year, starting at age 65 . He plans to pay for this with annual premiums from now until he turns 65 (so the first premium is today, the last premium is on his 64th birthday). The interest rate is $i=0.07$. The insurance company calculates that for this individual, $A_{65}=0.178416$ and $A_{48}=0.0716384$. What should the annual premiums be?

## Bonus Question

9. Consider a policy for a life currently aged $x$ which has a death benefit at the end of the year of death of $s_{\bar{k} \mid i}$ if the life dies between ages $x+k$ and $x+k+1$. What is the expected present value of this benefit?

Table 1: Select lifetable to be used for questions on this assignment

| $x$ | $l_{[x]}$ | $l_{[x]+1}$ | $l_{[x]+2}$ | $l_{[x]+3}$ | $x$ | $l_{[x]}$ | $l_{[x]+1}$ | $l_{[x]+2}$ | $l_{[x]+3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25 | 9998.75 | 9997.65 | 9996.30 | 9994.66 | 74 | 8987.73 | 8932.10 | 8862.49 | 8775.52 |
| 26 | 9997.00 | 9995.83 | 9994.40 | 9992.66 | 75 | 8897.04 | 8836.71 | 8761.27 | 8667.10 |
| 27 | 9995.14 | 9993.90 | 9992.38 | 9990.52 | 76 | 8798.69 | 8733.34 | 8651.66 | 8549.78 |
| 28 | 9993.16 | 9991.84 | 9990.22 | 9988.24 | 77 | 8692.13 | 8621.41 | 8533.09 | 8423.00 |
| 29 | 9991.05 | 9989.65 | 9987.92 | 9985.80 | 78 | 8576.81 | 8500.36 | 8404.95 | 8286.16 |
| 30 | 9988.81 | 9987.30 | 9985.46 | 9983.18 | 79 | 8452.13 | 8369.60 | 8266.68 | 8138.66 |
| 31 | 9986.40 | 9984.80 | 9982.82 | 9980.38 | 80 | 8317.52 | 8228.53 | 8117.67 | 7979.93 |
| 32 | 9983.83 | 9982.11 | 9979.99 | 9977.37 | 81 | 8172.36 | 8076.57 | 7957.35 | 7809.41 |
| 33 | 9981.07 | 9979.23 | 9976.95 | 9974.13 | 82 | 8016.08 | 7913.13 | 7785.15 | 7626.56 |
| 34 | 9978.11 | 9976.13 | 9973.68 | 9970.64 | 83 | 7848.11 | 7737.67 | 7600.54 | 7430.89 |
| 35 | 9974.93 | 9972.79 | 9970.16 | 9966.88 | 84 | 7667.89 | 7549.66 | 7403.05 | 7221.99 |
| 36 | 9971.50 | 9969.20 | 9966.36 | 9962.82 | 85 | 7474.92 | 7348.64 | 7192.27 | 6999.51 |
| 37 | 9967.80 | 9965.33 | 9962.25 | 9958.44 | 86 | 7268.77 | 7134.21 | 6967.86 | 6763.22 |
| 38 | 9963.81 | 9961.14 | 9957.82 | 9953.69 | 87 | 7049.07 | 6906.07 | 6729.62 | 6513.04 |
| 39 | 9959.50 | 9956.61 | 9953.02 | 9948.55 | 88 | 6815.55 | 6664.05 | 6477.46 | 6249.02 |
| 40 | 9954.84 | 9951.71 | 9947.82 | 9942.98 | 89 | 6568.09 | 6408.10 | 6211.48 | 5971.42 |
| 41 | 9949.79 | 9946.41 | 9942.19 | 9936.94 | 90 | 6306.70 | 6138.35 | 5931.96 | 5680.73 |
| 42 | 9944.32 | 9940.66 | 9936.08 | 9930.38 | 91 | 6031.59 | 5855.15 | 5639.41 | 5377.67 |
| 43 | 9938.39 | 9934.41 | 9929.45 | 9923.26 | 92 | 5743.19 | 5559.08 | 5334.61 | 5063.27 |
| 44 | 9931.96 | 9927.64 | 9922.25 | 9915.52 | 93 | 5442.15 | 5250.97 | 5018.61 | 4738.86 |
| 45 | 9924.97 | 9920.28 | 9914.42 | 9907.10 | 94 | 5129.44 | 4931.97 | 4692.79 | 4406.12 |
| 46 | 9917.37 | 9912.28 | 9905.91 | 9897.94 | 95 | 4806.33 | 4603.54 | 4358.89 | 4067.08 |
| 47 | 9909.11 | 9903.58 | 9896.65 | 9887.98 | 96 | 4474.39 | 4267.51 | 4018.96 | 3724.10 |
| 48 | 9900.13 | 9894.11 | 9886.57 | 9877.13 | 97 | 4135.60 | 3926.04 | 3675.44 | 3379.91 |
| 49 | 9890.36 | 9883.80 | 9875.59 | 9865.30 | 98 | 3792.25 | 3581.66 | 3331.11 | 3037.57 |
| 50 | 9879.71 | 9872.57 | 9863.63 | 9852.42 | 99 | 3447.02 | 3237.23 | 2989.05 | 2700.39 |
| 51 | 9868.12 | 9860.34 | 9850.59 | 9838.38 | 100 | 3102.90 | 2895.94 | 2652.63 | 2371.88 |
| 52 | 9855.48 | 9847.01 | 9836.39 | 9823.08 | 101 | 2763.19 | 2561.21 | 2325.37 | 2055.64 |
| 53 | 9841.72 | 9832.48 | 9820.90 | 9806.39 | 102 | 2431.39 | 2236.61 | 2010.90 | 1755.27 |
| 54 | 9826.71 | 9816.64 | 9804.02 | 9788.18 | 103 | 2111.15 | 1925.80 | 1712.81 | 1474.18 |
| 55 | 9810.34 | 9799.37 | 9785.60 | 9768.33 | 104 | 1806.12 | 1632.34 | 1434.48 | 1215.44 |
| 56 | 9792.49 | 9780.52 | 9765.51 | 9746.67 | 105 | 1519.82 | 1359.55 | 1178.94 | 981.65 |
| 57 | 9773.03 | 9759.97 | 9743.60 | 9723.05 | 106 | 1255.46 | 1110.36 | 948.70 | 774.71 |
| 58 | 9751.79 | 9737.56 | 9719.69 | 9697.28 | 107 | 1015.81 | 887.14 | 745.58 | 595.71 |
| 59 | 9728.63 | 9713.10 | 9693.62 | 9669.17 | 108 | 802.96 | 691.49 | 570.56 | 444.87 |
| 60 | 9703.36 | 9686.43 | 9665.17 | 9638.51 | 109 | 618.23 | 524.17 | 423.71 | 321.41 |
| 61 | 9675.80 | 9657.33 | 9634.15 | 9605.07 | 110 | 462.04 | 385.00 | 304.13 | 223.65 |
| 62 | 9645.73 | 9625.59 | 9600.31 | 9568.61 | 111 | 333.80 | 272.80 | 210.00 | 149.10 |
| 63 | 9612.94 | 9590.98 | 9563.42 | 9528.85 | 112 | 231.99 | 185.53 | 138.71 | 94.62 |
| 64 | 9577.18 | 9553.24 | 9523.19 | 9485.52 | 113 | 154.19 | 120.34 | 87.07 | 56.74 |
| 65 | 9538.19 | 9512.09 | 9479.35 | 9438.30 | 114 | 97.30 | 73.90 | 51.50 | 31.84 |
| 66 | 9495.69 | 9467.25 | 9431.58 | 9386.86 | 115 | 57.78 | 42.55 | 28.41 | 16.52 |
| 67 | 9449.37 | 9418.39 | 9379.54 | 9330.85 | 116 | 31.92 | 22.69 | 14.43 | 7.81 |
| 68 | 9398.90 | 9365.17 | 9322.87 | 9269.88 | 117 | 16.15 | 11.04 | 6.63 | 3.30 |
| 69 | 9343.95 | 9307.23 | 9261.20 | 9203.55 | 118 | 7.34 | 4.79 | 2.69 | 1.21 |
| 70 | 9284.12 | 9244.18 | 9194.11 | 9131.43 | 119 | 2.90 | 1.79 | 0.93 | 0.37 |
| 71 | 9219.03 | 9175.59 | 9121.17 | 9053.07 | 120 | 0.95 | 0.55 | 0.26 | 0.09 |
| 72 | 9148.24 | 9101.03 | 9041.91 | 8967.97 | 121 | 0.23 | 0.13 | 0.05 | 0.01 |
| 73 | 9071.30 | 9020.03 | 8955.85 | 8875.63 | 122 | 0.03 | 0.02 | 0.01 | 0.00 |

