ACSC/STAT 3720, Life Contingencies I
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Toby Kenney
Homework Sheet 2
Model Solutions

Basic Questions

1. *Using the select lifetable in Table 1, calculate:*

   (a) the probability that an individual aged 36 who was select 1 year ago dies within the next 8 years.

   The probability is ${}_8q_{[35]+1} = \frac{l_{[41]+3}}{l_{[35]+1}} = \frac{9936.94}{9972.79} = 0.9964 = 0.0036$.

   (b) the probability that an individual aged 42 who was select 6 years ago dies within the next 4 years.

   The probability is ${}_4q_{[36]+6} ={}_4 q_{[39]+3} = 1 - \frac{l_{[43]+3}}{l_{[39]+3}} = 1 - \frac{9923.36}{9948.55} = 1 - .9975 = 0.0025$.

   (c) the probability that an individual aged 33 who is select survives to age 65.

   This probability is given by $\frac{l_{[62]+3}}{l_{[33]}} = \frac{9568.61}{9981.07} = 0.9617$.

   (d) ${}_{3|6}q_{[35]+2}$

   This is the probability that an individual aged 37 who was select 2 years ago survives for 3 years, then dies within the following 6 years. It is calculated as the product ${}_{3|6}q_{[35]+2} ={}_3 p_{[35]+2}{}_6q_{[35]+5} = \frac{l_{[37]+3}-l_{[43]+3}}{l_{[35]+2}} = \frac{9958.44-9923.26}{9970.16} = 0.003529$.

2. *Using the select lifetable in Table 1, calculate the expected curtate future lifetime for*

   (a) a select individual aged 65

   The expected curtate future lifetime is given by

$$\sum_{n=1}^{\infty} {}_np_{[65]} = \frac{\sum_{n=1}^{\infty} l_{[65]+n}}{l_{[65]}}$$

   We evaluate this sum as

$$\frac{9512.09 + 9479.35 + 9438.30 + \cdots + 3.30 + 1.21 + 0.37 + 0.09 + 0.01}{9538.19} = 29.13$$

   (b) an individual aged 65 in the ultimate part of the model.

   This is given as

$$\sum_{n=1}^{\infty} {}_np_{[62]+3} = \frac{\sum_{n=1}^{\infty} l_{[62]+n+3}}{l_{[62]+3}}$$

   We evaluate this sum as

$$\frac{9528.85 + 9485.52 + 9438.30 + \cdots + 3.30 + 1.21 + 0.37 + 0.09 + 0.01}{9568.61} = 29.04$$

3. *Using the ultimate mortality model*

$$\mu_x = 0.000077 + 0.0000078 \times 1.099^x$$

and select mortality

$$\mu_{[x+s]} = 0.85^{2-s}\mu_{x+s}$$

calculate a select life table between ages 54 and 59 using radix 10,000. [You may use the approximation $q_{[x]+t} = \mu_{[x+0.5]+t}$.]

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$
54	9993.75	9983.53	9970.40
55	9979.02	9967.86	9953.52
56	9962.93	9950.74	9935.07
57	9945.36	9932.04	9914.91
58	9926.15	9911.60	9892.88
59	9905.16	9889.26	9868.80

Standard Questions

4. *An insurance policy is considering a new term insurance policy where individuals have the opportunity to renew their select status after 3 years of the policy and every 3 years thereafter. The appropriate lifetable is Table 1. The company finds that a select individual aged 38 has a probability 0.93 of being alive and select again 3 years later. What is the probability that an individual who is alive but not select again 3 years later (at age 41) survives to the end of the 10-year term (i.e. at age 48)?*

The probability that a select individual aged 38 is alive 3 years later is $\frac{9953.69}{9963.81} = 0.998984324$. The probability that a select individual aged 38 is alive 10 years later is $\frac{9907.10}{9963.81} = 0.994308402$. The probability that a select individual aged 38 is select again 3 years later and survives to the end of the 10-year term is $0.93\frac{9907.10}{9949.79} = 0.926009795$. Therefore, the probability that a select individual aged 38 is not select 3 years later, but survives to the end of the 10 year term is $0.994308402 - 0.926009795 = 0.0682986$. The probability that a select individual aged 38 is alive but not select 3 years later is $0.998984324 - 0.93 = 0.068984$. Therefore the probability that such an individual survives to the end of the 10-year term is $\frac{0.0682986}{0.068984} = 0.9901$.