

ACSC/STAT 3720, Life Contingencies I
 Winter 2018
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 Homework Sheet 2
 Model Solutions

Basic Questions

1. Using the select lifetable in Table 1, calculate:

(a) the probability that an individual aged 42 who was select 1 year ago dies within the next 4 years.

$${}^4q_{[41]+1} = 1 - {}_4p_{[41]+1} = 1 - \frac{l_{[41]+5}}{l_{[41]+1}} = 1 - \frac{l_{[43]+3}}{l_{[41]+1}} = 1 - \frac{9923.26}{9946.41} = 0.002327472927$$

(b) the probability that an individual aged 33 who was select 4 years ago dies within the next 3 years.

$${}^3q_{[29]+4} = 1 - {}_3p_{[29]+4} = 1 - \frac{l_{[29]+7}}{l_{[29]+4}} = 1 - \frac{l_{[33]+3}}{l_{[30]+3}} = 1 - \frac{9974.13}{9983.18} = 0.000906524775$$

(c) the probability that an individual aged 57 who is select survives to age 65.

$${}_8p_{[57]} = \frac{l_{[57]+8}}{l_{[57]}} = \frac{l_{[62]+3}}{l_{[57]}} = \frac{9568.61}{9773.03} = 0.979083252584$$

(d) ${}_9p_{[34]+2}$

$${}_9p_{[34]+2} = \frac{l_{[34]+11}}{l_{[34]+2}} = \frac{l_{[42]+3}}{l_{[34]+2}} = \frac{9930.38}{9973.68} = 0.995658573365$$

2. An individual's mortality follows the select lifetable in Table 1. You are given that for this lifetable, the expected curtate future lifetime for an individual at age 65 in the ultimate part of the model is 29.04. Calculate the expected curtate future lifetime for

(a) a select individual aged 62

We calculate

$$\begin{aligned} p_{[62]} &= \frac{9625.59}{9645.73} = 0.997912029468 \\ {}_2p_{[62]} &= \frac{9600.31}{9645.73} = 0.995291180657 \\ {}_3p_{[62]} &= \frac{9568.61}{9645.73} = 0.992004752362 \end{aligned}$$

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66	74	8987.73	8932.10	8862.49	8775.52
26	9997.00	9995.83	9994.40	9992.66	75	8897.04	8836.71	8761.27	8667.10
27	9995.14	9993.90	9992.38	9990.52	76	8798.69	8733.34	8651.66	8549.78
28	9993.16	9991.84	9990.22	9988.24	77	8692.13	8621.41	8533.09	8423.00
29	9991.05	9989.65	9987.92	9985.80	78	8576.81	8500.36	8404.95	8286.16
30	9988.81	9987.30	9985.46	9983.18	79	8452.13	8369.60	8266.68	8138.66
31	9986.40	9984.80	9982.82	9980.38	80	8317.52	8228.53	8117.67	7979.93
32	9983.83	9982.11	9979.99	9977.37	81	8172.36	8076.57	7957.35	7809.41
33	9981.07	9979.23	9976.95	9974.13	82	8016.08	7913.13	7785.15	7626.56
34	9978.11	9976.13	9973.68	9970.64	83	7848.11	7737.67	7600.54	7430.89
35	9974.93	9972.79	9970.16	9966.88	84	7667.89	7549.66	7403.05	7221.99
36	9971.50	9969.20	9966.36	9962.82	85	7474.92	7348.64	7192.27	6999.51
37	9967.80	9965.33	9962.25	9958.44	86	7268.77	7134.21	6967.86	6763.22
38	9963.81	9961.14	9957.82	9953.69	87	7049.07	6906.07	6729.62	6513.04
39	9959.50	9956.61	9953.02	9948.55	88	6815.55	6664.05	6477.46	6249.02
40	9954.84	9951.71	9947.82	9942.98	89	6568.09	6408.10	6211.48	5971.42
41	9949.79	9946.41	9942.19	9936.94	90	6306.70	6138.35	5931.96	5680.73
42	9944.32	9940.66	9936.08	9930.38	91	6031.59	5855.15	5639.41	5377.67
43	9938.39	9934.41	9929.45	9923.26	92	5743.19	5559.08	5334.61	5063.27
44	9931.96	9927.64	9922.25	9915.52	93	5442.15	5250.97	5018.61	4738.86
45	9924.97	9920.28	9914.42	9907.10	94	5129.44	4931.97	4692.79	4406.12
46	9917.37	9912.28	9905.91	9897.94	95	4806.33	4603.54	4358.89	4067.08
47	9909.11	9903.58	9896.65	9887.98	96	4474.39	4267.51	4018.96	3724.10
48	9900.13	9894.11	9886.57	9877.13	97	4135.60	3926.04	3675.44	3379.91
49	9890.36	9883.80	9875.59	9865.30	98	3792.25	3581.66	3331.11	3037.57
50	9879.71	9872.57	9863.63	9852.42	99	3447.02	3237.23	2989.05	2700.39
51	9868.12	9860.34	9850.59	9838.38	100	3102.90	2895.94	2652.63	2371.88
52	9855.48	9847.01	9836.39	9823.08	101	2763.19	2561.21	2325.37	2055.64
53	9841.72	9832.48	9820.90	9806.39	102	2431.39	2236.61	2010.90	1755.27
54	9826.71	9816.64	9804.02	9788.18	103	2111.15	1925.80	1712.81	1474.18
55	9810.34	9799.37	9785.60	9768.33	104	1806.12	1632.34	1434.48	1215.44
56	9792.49	9780.52	9765.51	9746.67	105	1519.82	1359.55	1178.94	981.65
57	9773.03	9759.97	9743.60	9723.05	106	1255.46	1110.36	948.70	774.71
58	9751.79	9737.56	9719.69	9697.28	107	1015.81	887.14	745.58	595.71
59	9728.63	9713.10	9693.62	9669.17	108	802.96	691.49	570.56	444.87
60	9703.36	9686.43	9665.17	9638.51	109	618.23	524.17	423.71	321.41
61	9675.80	9657.33	9634.15	9605.07	110	462.04	385.00	304.13	223.65
62	9645.73	9625.59	9600.31	9568.61	111	333.80	272.80	210.00	149.10
63	9612.94	9590.98	9563.42	9528.85	112	231.99	185.53	138.71	94.62
64	9577.18	9553.24	9523.19	9485.52	113	154.19	120.34	87.07	56.74
65	9538.19	9512.09	9479.35	9438.30	114	97.30	73.90	51.50	31.84
66	9495.69	9467.25	9431.58	9386.86	115	57.78	42.55	28.41	16.52
67	9449.37	9418.39	9379.54	9330.85	116	31.92	22.69	14.43	7.81
68	9398.90	9365.17	9322.87	9269.88	117	16.15	11.04	6.63	3.30
69	9343.95	9307.23	9261.20	9203.55	118	7.34	4.79	2.69	1.21
70	9284.12	9244.18	9194.11	9131.43	119	2.90	1.79	0.93	0.37
71	9219.03	9175.59	9121.17	9053.07	120	0.95	0.55	0.26	0.09
72	9148.24	9101.03	9041.91	8967.97	121	0.23	0.13	0.05	0.01
73	9071.30	9020.03	8955.85	8875.63	122	0.03	0.02	0.01	0.00

This gives that the curtate expected lifetime is $0.997912029468+0.995291180657+0.992004752362+0.992004752362 \times 29.04 = 31.7930259711$.

(b) an individual aged 59 who was select 1 year ago.

We calculate

$$\begin{aligned} p_{[58]+1} &= \frac{9719.69}{9737.56} = 0.998164838009 \\ 2p_{[58]+1} &= \frac{9697.28}{9737.56} = 0.995863440123 \\ 3p_{[58]+1} &= \frac{9669.17}{9737.56} = 0.992976679990 \\ 4p_{[58]+1} &= \frac{9638.51}{9737.56} = 0.989828047273 \\ 5p_{[58]+1} &= \frac{9605.07}{9737.56} = 0.986393922091 \\ 6p_{[58]+1} &= \frac{9568.61}{9737.56} = 0.982649657614 \end{aligned}$$

This gives that the curtate expected lifetime is $0.998164838009+0.995863440123+0.992976679990+0.989828047273+0.986393922091+0.982649657614+0.982649657614 \times 29.04 = 34.4820226422$

3. Using the ultimate mortality model

$$\mu_x = 0.000075 + 0.0000139 \times 1.122^x$$

and select mortality

$$\mu_{[x]+s} = 0.85^{3-s} \mu_{x+s}$$

for the selection period of 3 years, calculate a select life table between ages 38 and 40 using radix 10,000. [You may use the approximation $q_{[x]+t} = \mu_{[x+0.5]+t}$.]

Using the approximation $q_{[x]+t} = \mu_{[x+0.5]+t}$ gives the ultimate mortality model

$$\begin{aligned} l_{38} &= 10000.00 \\ l_{39} &= 10000.00 (1 - (0.000075 + 0.0000139 \times 1.122^{38.5})) = 9987.56201107 \\ l_{40} &= 9987.56201107 (1 - (0.000075 + 0.0000139 \times 1.122^{39.5})) = 9973.71533142 \\ l_{41} &= 9973.71533142 (1 - (0.000075 + 0.0000139 \times 1.122^{40.5})) = 9958.2921553 \\ l_{42} &= 9958.2921553 (1 - (0.000075 + 0.0000139 \times 1.122^{41.5})) = 9941.10522991 \\ l_{43} &= 9941.10522991 (1 - (0.000075 + 0.0000139 \times 1.122^{41.5})) = 9923.94796728 \end{aligned}$$

This gives us the final column in the lifetable

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
38	??	??	??	9958.29
39	??	??	??	9941.11
40	??	??	??	9923.95

We now work backwards from the entry at the end of each row. For example,

$$l_{[38]+2} = \frac{9958.2921553}{(1 - 0.85(0.000075 + 0.0000139 \times 1.122^{40.5}))} = 9971.3988101$$

$$l_{[38]+1} = \frac{9971.3988101}{(1 - 0.85^2(0.000075 + 0.0000139 \times 1.122^{39.5}))} = 9981.39686071$$

$$l_{[38]} = \frac{9981.39686071}{(1 - 0.85^3(0.000075 + 0.0000139 \times 1.122^{38.5}))} = 9989.02695892$$

So we now have the following:

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
38	9990.37	9981.40	9971.40	9958.29
39	??	??	??	9941.11
40	??	??	??	9923.95

Applying the same procedure in the other rows gives the complete lifetable:

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
38	9989.03	9981.40	9971.40	9958.29
39	9975.34	9966.85	9955.71	9941.11
40	9960.09	9950.63	9938.23	9921.95

Standard Questions

4. An insurance company is reviewing its policies, and discovers that due to an administrative error, it sold a policy for a select life aged 32 to an individual who failed the underwriting. If the probability of a random life aged 32 passing the underwriting is 0.92. What is the probability that this policyholder dies within 5 years from the start of the policy? [Hint: Compare select and ultimate mortality for age 32.]

The probability of a random individual aged 32 surviving for 5 years is the survival probability from the ultimate part of the model, namely ${}_5p_{32} = \frac{l_{37}}{l_{32}} = \frac{9970.64}{9985.80} = 0.998481844219$ while the probability that a select individual survives is ${}_5p_{[32]} = \frac{l_{37}}{l_{[32]}} = \frac{9970.64}{9983.83} = 0.998678863723$. If we let p be the probability of this non-select life surviving for 5 years, then we have

$$\begin{aligned} 0.998481844219 &= 0.92 \times 0.998678863723 + 0.08p \\ 0.08p &= 0.998481844219 - 0.92 \times 0.998678863723 \\ &= 0.079697289594 \\ p &= \frac{0.079697289594}{0.08} \\ &= 0.996216119925 \end{aligned}$$

5. An insurance company has used Makeham's formula with a constant factor to discount for selected lives — that is $\mu_{[x]+s} = D^{3-s}\mu_{x+s}$ to construct a lifetable for female smokers. The lifetable is given below.

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
45	9891.82	9817.89	9721.40	9595.71
46	9758.74	9677.21	9570.86	9432.44
47	9611.91	9522.12	9405.06	9252.86
48	9450.12	9351.37	9222.74	9055.69
49	9272.11	9163.70	9022.61	8839.64
50	9076.58	8957.80	8803.39	8603.48

Find the parameters used in the model to produce the table. [It was constructed using the approximation $q_{[x]+s} = \mu_{[x+0.5]+s}$.]

We first calculate $q_{48} = 1 - \frac{9432.44}{9595.71} = 0.017014895198$, and $q_{[46]+2} = 1 - \frac{9432.44}{9570.86} = 0.014462650169$.

This gives us $D = \frac{0.014462650169}{0.017014895198} = 0.85$. Next we need to estimate A , B and C . We have

$$\begin{aligned}
 A + BC^{48.5} &= 0.017014895198 \\
 A + BC^{49.5} &= 0.019038552061 \\
 A + BC^{50.5} &= 0.021309087136 \\
 BC^{48.5}(C - 1) &= 0.019038552061 - 0.017014895198 = 0.002023656863 \\
 BC^{49.5}(C - 1) &= 0.021309087136 - 0.019038552061 = 0.002270535075 \\
 C &= \frac{0.002270535075}{0.002023656863} \\
 &= 1.12199608368 \\
 B &= \frac{0.002270535075}{0.12199608368 \times 1.12199608368^{49.5}} = 0.0000624
 \end{aligned}$$

[There are many other ways we could calculate these parameters from the table. Because the numbers in the lifetable are rounded, there might be slight differences between the parameters we get using different methods.]