ACSC/STAT 4703, Actuarial Models II
Fall 2016
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Homework Sheet 5
Due: Friday 4th November: 10:30 PM

Basic Questions

1. An insurance company is modelling claim data as following a Pareto distribution with $\alpha = 4$. It collects the following sample of claims:

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   18.0  52.1  67.5  89.4  99.6  131.0  153.5  161.0  174.4  223.1
   244.5  261.6  278.2  282.4  290.1  296.2  321.0  368.7  370.1
   382.8  412.7  431.2  645.6  664.0  1915.5
   ```

   The MLE for θ is 1119.3399. Graphically compare this empirical distribution with the best fitting Pareto distribution with $\alpha = 4$. Include the following plots:

   (a) Comparisons of $F(x)$ and $F^*(x)$

   (b) Comparisons of $f(x)$ and $f^*(x)$

   (c) A plot of $D(x)$ against x.

   (d) A p-p plot of $F(x)$ against $F^*(x)$.

2. For the data in Question 1, calculate the following test statistics for the goodness of fit of the Pareto distribution with $\alpha = 4$ and $\theta = 1119.3399$ using:

   (a) The Kolmogorov-Smirnov test.

   (b) The Anderson-Darling test.

   (c) The chi-square test, dividing into the intervals 0–200, 200–400, and more than 400.

3. For the data in Question 1, perform a likelihood ratio test to determine whether a Pareto distribution with fixed $\alpha = 4$, or a Pareto distribution with α freely estimated is a better fit for the data. [The MLE for the general Pareto distribution is $\alpha = 22.49267$ and $\theta = 7159.3127$.]
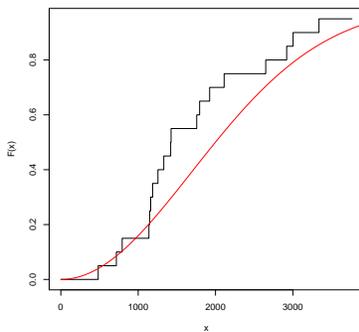
Standard Questions

4. An insurance company collects a sample of 20 past claims, and attempts to fit a distribution to the claims. Based on experience with other claims, the company believes that a Weibull distribution with $\tau = 2$ and $\theta = 2,400$

may be appropriate to model these claims. It constructs the following p-p plot to compare the sample to this distribution:
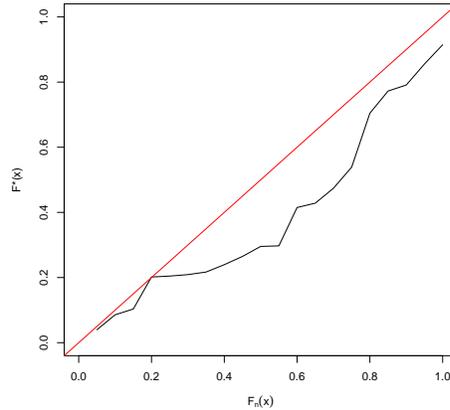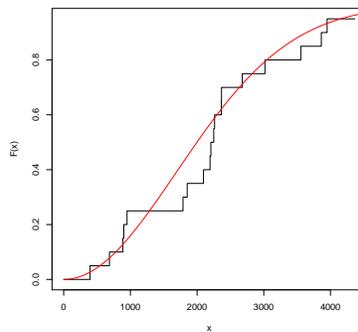

(a) How many of the points in their sample were less than 2,400?

(b) Which of the following statements best describes the fit of the Weibull distribution to the data:

(i) The Weibull distribution assigns too much probability to high values and too little probability to low values.

(ii) The Weibull distribution assigns too much probability to low values and too little probability to high values.

(iii) The Weibull distribution assigns too much probability to tail values and too little probability to central values.

(iv) The Weibull distribution assigns too much probability to central values and too little probability to tail values.

(c) Which of the following plots shows the empirical distribution function? Justify your answer.

(i)   (ii)   (iii)