ACSC/STAT 4703, Actuarial Models II
Fall 2017
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Sample Final Examination

This Sample examination has more questions than the actual final, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. An insurance company sells home insurance. It estimates that the standard deviation of the aggregate annual claim is \$5,326 and the mean is \$1,804.

   (a) How many years history are needed for an individual or group to be assigned full credibility? (Use $r = 0.05$, $p = 0.95$.) [5 mins.]

   (b) What is the Credibility premium, using limited fluctuation credibility, for an individual who has claimed a total of \$42,381 in the past 19 years? [5 mins.]

2. For a car insurance policy, the book premium for claim severity is \$2,300. An individual has made 7 claims in the past 12 years, with average claim severity \$1,074. Calculate the credibility estimate for claim severity for this individual using limited fluctuation credibility, if the standard for full credibility is:

   (a) 157 claims. [5 mins.]

   (b) 284 years. [5 mins.]

3. A worker's compensation insurance company classifies workplaces as "safe" or "hazardous". Claims from hazardous workplaces follow a Gamma distribution with $\alpha = 0.1021749$, $\theta = 1066798$ (mean \$109,000 and standard deviation \$341,000). Claims from safe workplaces follow a Gamma distribution with $\alpha = 0.01209244$, $\theta = 2646281$ (mean \$32,000 and standard deviation \$261,000). 94% of workplaces are classified as safe.

   [You may need the following values:

   $$\Gamma(0.01209244) = 82.13091$$
   $$\Gamma(0.1021749) = 9.302457$$

   (a) Calculate the expectation and variance of claim size for a claim from a randomly chosen workplace. [5 mins.]

   (b) The last 2 claims from a particular workplace are \$488,200 and \$17,400. Calculate the expectation and variance for the next claim size from this workplace. [10 mins.]

4. An insurance company sets the book pure premium for its home insurance at \$791. The expected process variance is 6,362,000 and the variance of hypothetical means is 341,200. If an individual has no claims over the last 8 years, calculate the credibility premium for this individual's next year's insurance using the Bühlmann model. [5 mins.]

5. An insurance company is reviewing the premium for an individual with the following past claim history:

Year	1	2	3	4	5
Exposure	0.2	1	1	0.4	0.8
Aggregate claims	0	\$2,592	0	\$147	\$1,320

   The usual premium per unit of exposure is \$2,700. The expected process variance is 123045 and the variance of hypothetical means is 36403 (both per unit of exposure). Calculate the credibility premium for this individual if she has 0.6 units of exposure in year 6. [10 mins.]

6. An insurance company has 3 years of past history on a homeowner, denoted X_1, X_2, X_3. Because the individual moved house at the end of the second year, the third year has a different mean and variance, and is not as correlated with the other two years. It has the following

$$\begin{aligned}
\mathbb{E}(X_1) &= 1,322 & \mathrm{Var}(X_1) &= 226,000 \\
\mathbb{E}(X_2) &= 1,322 & \mathrm{Var}(X_2) &= 226,000 \\
\mathbb{E}(X_3) &= 4,081 & \mathrm{Var}(X_3) &= 1,108,000 \\
\mathbb{E}(X_4) &= 4,081 & \mathrm{Var}(X_4) &= 1,108,000 \\
\mathrm{Cov}(X_1, X_2) &= 214 & \mathrm{Cov}(X_1, X_3) &= 181 \\
\mathrm{Cov}(X_2, X_3) &= 181 & \mathrm{Cov}(X_1, X_4) &= 181 \\
\mathrm{Cov}(X_2, X_4) &= 181 & \mathrm{Cov}(X_3, X_4) &= 861
\end{aligned}$$

It uses a formula $\hat{X_4} = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$ to calculate the credibility premium in the fourth year. Calculate the values of α_0, α_1, α_2 and α_3. [15 mins.]

7. An insurance company has the following previous data on aggregate claims:

Policyholder	Year 1	Year 2	Year 3	Year 4	Mean	Variance
1	1,210	246	459	1,461	944.00	340158.00
2	0	0	0	0	0.00	0.00
3	0	2,185	0	0	548.25	1202312.25
4	809	0	0	1,725	633.50	674939.00
5	0	0	0	0	0.00	0.00

Calculate the Bühlmann credibility premium for policyholder 3 in Year 5. [15 mins.]

8. An insurance company collects the following claim frequency data for 7,000 customers insured for the past 3 years:

No. of claims	Frequency
0	1,491
1	2,461
2	1,810
3	831
4	302
5	72
6	30
7	2
8	1
> 8	0

It assumes that the number of claims an individual makes in a year follows a Poisson distribution with parameter Λ, which may vary between individuals.

Find the credibility estimate for the expected number of claims per year for an individual who has made 4 claims in the past 3 years. [15 mins.]

9. An insurance company starts a new line of insurance in 2016, and collects a total of $1,900,000 in premiums that year, and the estimated incurred losses for accident year 2016 are $1,384,000. Half of the premium payments are made at the beginning of the year, and the other half are uniformly distributed over the year. An actuary is using this data to estimate rates for premium year 2018. Claims are subject to 4% inflation per year. By what percentage should premiums increase from 2016 in order to achieve a loss ratio of 0.75? [15 mins]

10. An insurer classifies policies into three classes — single, couple and family. The experience from policy year 2016 is:

Class	Current differential	Earned premiums	Loss payments
Single	0.74	4,740	3,940
Couple	0.93	4,490	3,880
Family	1	5,670	4,930

The base premium was $420. Claim amounts are subject to 4% annual inflation. If the expense ratio is 25%, calculate the new premiums for each age class for policy year 2018. [15 mins]

11. An insurer has different premiums for personal and commercial vehicles. Its experience for accident year 2016 is given below. There was a rate change on 1st August 2015, which affects some policies in 2016.

Type	Differential before rate change	Current differential	Earned premiums	Loss payments
Personal	1	1	11,300	9,800
Commercial	1.51	1.67	7,600	6,300

Before the rate change, the base premium was $950. The current base premium is $1,020. Assuming that policies were sold uniformly over the year, calculate the new premimums for policy year 2018 assuming 6% annual inflation and a permissible loss ratio of 0.75. [15 mins]

12. An insurance company has the following data on its policies:

Policy limit	Losses Limited to			
	20,000	50,000	100,000	500,000
20,000	1,400,000			
50,000	7,540,000	8,010,000		
100,000	22,600,000	24,100,000	28,700,000	
500,000	5,900,000	6,220,000	6,650,000	6,920,000

Use this data to calculate the ILF from $20,000 to $500,000 using

(a) The direct ILF estimate. [5 mins]

(b) The incremental method. [5 mins]

13. For a certain line of insurance, the loss amount per claim follows a Pareto distribution with $\alpha = 4$. If the policy has a deductible per loss set at 0.1θ and a policy limit set at 2θ, by how much will the expected payment per loss increase if there is inflation of 5%? [10 mins]

14. An insurance company charges a risk charge equal to the square of the average loss amount, divided by 100,000. It has the following data on a set of 1,200 claims from policies with limit $1,000,000.

Losses Limited to	50,000	100,000	500,000	1,000,000
Total claimed	16,700,000	20,880,000	27,030,000	32,410,000

Calculate the ILF from $100,000 to $1,000,000. [10 mins]

15. An insurer calculates the ILF on the pure premium from \$1,000,000 to \$2,000,000 on a particular policy is 1.092. A reinsurer offers excess-of-loss reinsurance of \$1,000,000 over \$1,000,000 for a loading of 30%. The original insurer uses a loading of 20% on policies with limit \$1,000,000. If the insurer buys the excess-of-loss reinsurance, what is the loading on its premium for policies with a limit of \$2,000,000? [10 mins]