ACSC/STAT 4703, Actuarial Models II
FALL 2017
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Sample Midterm Examination

This Sample examination has more questions than the actual midterm, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

Here are some values of the Gamma distribution function with $\theta = 1$ that will be needed for this examination:

x	α	$F(x)$
245	255	0.2697208
$\left(\frac{7.5}{12}\right)^3$	$\frac{4}{3}$	0.1117140
$\left(\frac{9.5}{12}\right)^3$	$\frac{4}{3}$	0.2507382
2.5	1	0.917915
2.5	2	0.7127025
2.5	3	0.4561869
2.5	4	0.2424239

1. Loss amounts follow an exponential distribution with $\theta = 60,000$. The distribution of the number of losses is given in the following table:

Number of Losses	Probability
0	0.04
1	0.54
2	0.27
3	0.15

   Assume all losses are independent and independent of the number of losses. The insurance company buys excess-of-loss reinsurance on the part of the loss above $150,000. Calculate the expected payment for this excess-of-loss reinsurance.

2. Aggregate payments have a compund distribution. The frequency distribution is negative binomial with $r = 4$ and $\beta = 12$. The severity distribution is a Gamma distribution with $\alpha = 8$ and $\theta = 3000$. Use a normal approximation to aggregate payments to estimate the probability that aggregate payments are more than $2,000,000.

3. Claim frequency follows a negative binomial distribution with $r = 5$ and $\beta = 2.9$. Claim severity (in thousands) has the following distribution:

Severity	Probability
0	0
1	0.600
2	0.220
3	0.166

Use the recursive method to calculate the exact probability that aggregate claims are at least 4.

4. Using an arithmetic distribution ($h = 1$) to approximate a Weibull distribution with $\tau = 3$ and $\theta = 12$, calculate the probability that the value is between 3.5 and 8.5, for the approximation using:

   (a) The method of rounding.

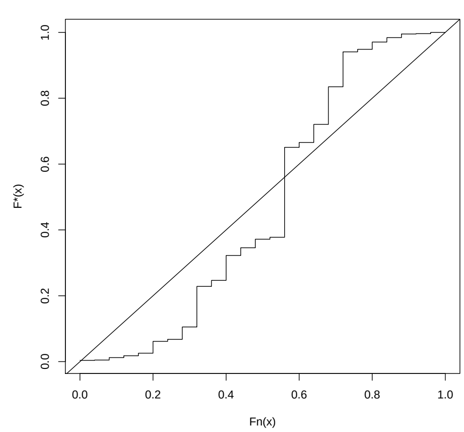
   (b) The method of local moment matching, matching 1 moment on each interval. [$\Gamma\left(\frac{4}{3}\right) = 0.8929795$.]

5. An insurance company has the following portfolio of auto insurance policies:

Type of driver	Number	Probability claim	mean of claim	standard deviation
Good driver	600	0.02	\$2,500	\$2,000
Average driver	1400	0.06	\$3,800	\$3,200
Bad driver	500	0.13	\$7,000	\$3,600

   Calculate the cost of reinsuring losses above \$5,000,000, if the loading on the reinsurance premium is one standard deviation above the expected claim payment on the reinsurance policy, using a Pareto approximation for the aggregate losses on this portfolio.

6. An insurance company collects a sample of 25 past claims, and attempts to fit a Pareto distribution to the claims. Based on experience with other claims, the company believes that a Pareto distribution with $\alpha = 3.5$ and $\theta = 4,600$ may be appropriate to model these claims. It constructs the following p-p plot to compare the sample to this distribution:



   (a) How many of the points in their sample were less than 1,200? [5 mins.]

   (b) Which of the following statements best describes the fit of the Pareto distribution to the data: [5 mins.]

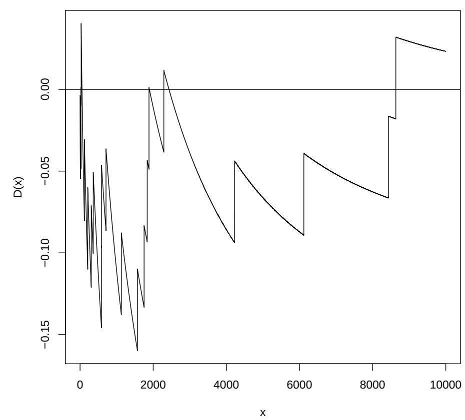
(i) The Pareto distribution assigns too much probability to high values and too little probability to low values.

(ii) The Pareto distribution assigns too much probability to low values and too little probability to high values.

(iii) The Pareto distribution assigns too much probability to tail values and too little probability to central values.

(iv) The Pareto distribution assigns too much probability to central values and too little probability to tail values.

7. An insurance company collects a sample of 20 claims. Based on previous experience, it believes these claims might follow a Weibull distribution with $\tau = 0.6$ and a known value of θ. To test this, it obtains a plot of $D(x)$.



(a) Which of the following is the value of θ used in the plot: [5 mins.]

(i) 800

(ii) 1,100

(iii) 2,200

(iv) 3,500

(b) Which of the following statements best describes the fit of the Weibull distribution to the data: [5 mins.]

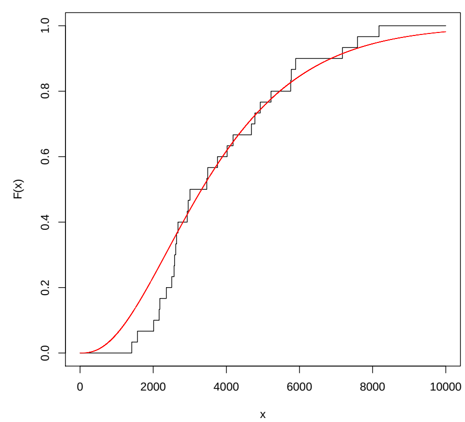
(i) The Weibull distribution assigns too much probability to high values and too little probability to low values.

(ii) The Weibull distribution assigns too much probability to low values and too little probability to high values.

(iii) The Weibull distribution assigns too much probability to tail values and too little probability to central values.

(iv) The Weibull distribution assigns too much probability to central values and too little probability to tail values.

8. An insurance company collects a sample of 30 claims. Based on previous experience, it believes these claims might follow a gamma distribution with $\alpha = 2.7$ and $\theta = 1400$. To test this, it compares plots of $F_n(x)$ and $F_*(x)$.



(a) Which of the following is the value of the Kolmogorov-Smirnov statistic for this model and this data [5 mins.]

(i) 0.0102432

(ii) 0.0450353

(iii) 0.0924252

(iv) 0.1678255

(b) Which of the following statements best describes the fit of the Gamma distribution to the data: [5 mins.]

(i) The Gamma distribution assigns too much probability to high values and too little probability to low values.

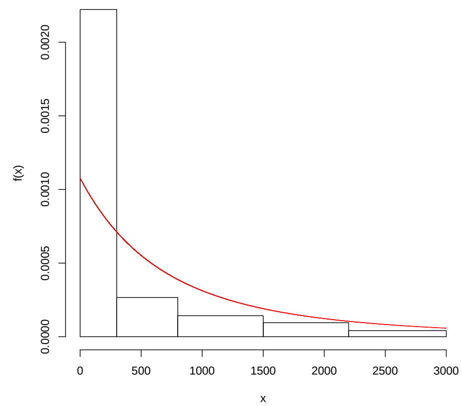
(ii) The Gamma distribution assigns too much probability to low values and too little probability to high values.

(iii) The Gamma distribution assigns too much probability to tail values and too little probability to central values.

(iv) The Gamma distribution assigns too much probability to central values and too little probability to tail values.

9. An insurance company collects a sample of 30 past claims, and attempts to fit a Pareto distribution to the claims. Based on experience with other claims, the company believes that a Pareto distribution with $\alpha = 2.8$ and

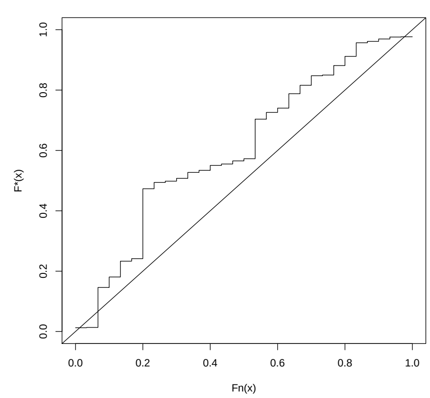
$\theta = 2,600$ may be appropriate to model these claims. It compares the density functions in the following plot:



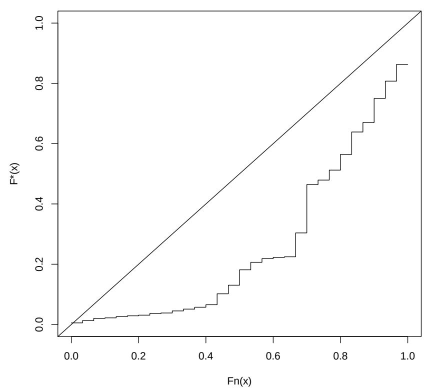
(a) How many data points in the sample were between 1500 and 3000? [5 mins.]

(b) Which of the following plots is the p-p plot for this data and model? [10 mins.]

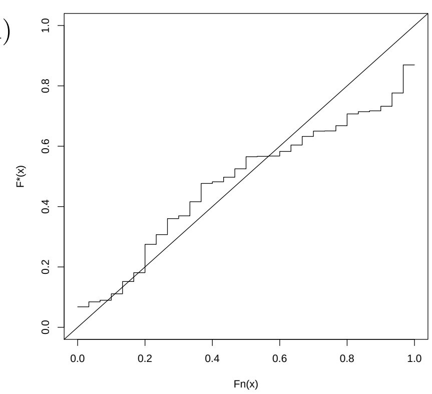
(i)



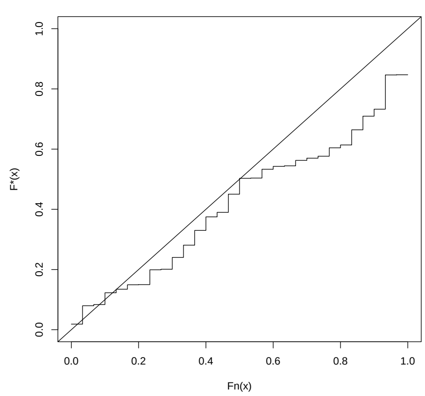
(ii)



(iii)



(iv)



10. An insurance company collects the following sample:

    `2.31 8.65 35.29 42.27 151.51 194.99 523.50 1262.01 1402.72 6063.74`

They model this as following a Pareto distribution with $\alpha = 2$ and $\theta = 2000$. Calculate the Kolmogorov-Smirnov statistic for this model and this data. [10 mins.]

11. An insurance company collects the following sample:

```
0.27 2.03 9.89 16.96 28.38 236.46 268.36 453.19 633.26
718.68 1414.59 1588.19 2535.69 4937.93 5431.13
```

They model this as following a gamma distribution with $\alpha = 0.4$ and $\theta = 6000$. Calculate the Anderson-Darling statistic for this model and this data. [10 mins.]

You are given the following values of the Gamma distribution used in the model:

x	$F(x)$	$\log(F(x))$	$\log(1-F(x))$
0.27	0.02056964	-3.8839392	-0.02078414
2.03	0.04609387	-3.0770753	-0.04719001
9.89	0.08680820	-2.4440542	-0.09080935
16.96	0.10767291	-2.2286572	-0.11392253
28.38	0.13222244	-2.0232696	-0.14181987
236.46	0.30572308	-1.1850755	-0.36488438
268.36	0.32111513	-1.1359556	-0.38730373
453.19	0.39258278	-0.9350079	-0.49853938
633.26	0.44506880	-0.8095264	-0.58891114
718.68	0.46633756	-0.7628455	-0.62799177
1414.59	0.59250242	-0.5234003	-0.89772028
1588.19	0.61583950	-0.4847689	-0.95669484
2535.69	0.71295893	-0.3383315	-1.24812996
4937.93	0.84646394	-0.1666877	-1.87381984
5431.13	0.86352967	-0.1467270	-1.99164807

12. An insurance company collects the following sample:

```
105.13 304.10 323.11 359.09 360.43 368.63 413.47 448.81
606.88 612.58 930.35 1002.37 1161.78 1205.25 5585.37
```

They want to decide whether this data is better modeled as following an inverse gamma distribution, or an inverse exponential distribution. They calculate that the MLEs for the inverse gamma distribution as $\alpha = 1.695545$ and $\theta = 705.7664$, and the MLE for the inverse exponential distribution as $\theta = 416.2476$. They also calculate, for this data that $\sum_{i=1}^{15} \log(x_i) = 95.31415$ and $\sum_{i=1}^{15} \frac{1}{x_i} = 0.03603625$, and that $\Gamma(1.695545) = 0.9078021$. You are given the following table of critical values for the chi-squared distribution at the 5% significance level. Indicate in your answer which critical value you are using. [15 mins.]

Degrees of Freedom	95% critical value
1	3.841459
2	5.991465
3	7.814728
4	9.487729
5	11.070498

13. An insurance company collects the following sample:

```
0.1  0.2  0.3  2.1  16.8  28.4  45.7  53.5  74.2  99.5  159.3
183.5  206.3  273.9  461.9  482.9  1118.5  1444.7  2084.3  3984.8
```

    They want to decide whether this data is better modeled as following an inverse exponential distribution or a Weibull distribution. They calculate that the MLE for the inverse exponential distribution is $\theta = 1.052901$, and the corresponding likelihood is -183.51. They also calculate that for the Weibull distribution, the MLE is $\tau = 0.48$, $\theta = 255.2235$. The log-likelihood is therefore -141.8325. Use AIC and BIC to determine which distribution is a better fit for the data. [5 mins.]

14. An insurance company collects the following data sample on claims data

Claim Amount	Number of Claims
Less than \$5,000	1,026
\$5,000–\$10,000	850
\$10,000–\$20,000	1,182
\$20,000–\$50,000	942
More than \$50,000	573

    Its previous experience suggests that the distribution should be modelled as following a Pareto distribution with $\alpha = 3$ and $\theta = 28,000$. Perform a chi-squared test to determine whether this distribution is a good fit for the data at the 95% level. [10 mins.]

    You may use the following critical values for the chi-squared distribution:

Degrees of Freedom	95% critical value
1	3.841459
2	5.991465
3	7.814728
4	9.487729
5	11.070498

15. A homeowner's house is valued at \$560,000. However, the home is insured only to a value of \$360,000. The insurer requires 80% coverage for full insurance. The home sustains \$6,000 of fire damage. The deductible is \$5,000, decreasing linearly to zero for losses of \$8,000. How much does the insurer reimburse?

16. An auto insurance company uses an expected loss ratio of 0.81. In accident year 2014, the earned premiums were \$1,420,000. In 2014, the insurance

company made a total of \$189,300 in loss payments for accident year 2014, a total of \$152,500 in 2015, and a total of \$239,600 in 2016. What loss reserves should the company hold for this accident year at the end of 2016.

17. The following table shows the cumulative losses (in thousands) on claims from one line of business of an insurance company over the past 6 years.

	Development year					
Accident year	0	1	2	3	4	5
2011	751	1,022	1,448	1,133	1,473	1,493
2012	1,337	1,297	1,460	1,537	1,679	
2013	1,250	1,624	1,815	1,860		
2014	1,325	1,512	1,685			
2015	1,471	1,536				
2016	2,036					

Using the average for calculating loss development factors, esimate the total reserve needed for payments to be made in 2018 using.

(a) The loss development triangle method

(b) The Bornhuetter-Fergusson method. The expected loss ratio is 0.76 and the earned premiums in each year are given in the following table:

Year	Earned Premiums (000's)
2011	1943
2012	2430
2013	2623
2014	2804
2015	3356
2016	3673

18. An actuary is reviewing the following loss development triangles:

No. of closed claims

Acc.	Development Year				Ult.
Year	0	1	2	3	
2013	482	481	579	636	660
2014	672	677	786		802
2015	657	734			823
2016	745				963

Total paid losses on closed claims (000's)

Acc.	Development Year			
Year	0	1	2	3
2013	1176	1163	1284	1372
2014	1130	1356	1292	
2015	1409	1507		
2016	2262			

(a) Calculate tables of percentage of claims closed and cumulative average losses.

(b) Adjust the total paid losses to use the current disposal rate.