ACSC/STAT 4703, Actuarial Models II
Fall 2017
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Homework Sheet 7
Model Solutions

Basic Questions

1. *An insurance company has the following data on its policies:*

Policy limit	*Losses Limited to*			
	20,000	*50,000*	*100,000*	*500,000*
20,000	*1,000,000*			
50,000	*7,040,000*	*8,010,000*		
100,000	*23,600,000*	*28,400,000*	*30,700,000*	
500,000	*5,050,000*	*5,340,000*	*5,500,000*	*5,930,000*

Use this data to calculate the ILF from \$20,000 to \$500,000 using

(a) The direct ILF estimate.

For policies with a limit of \$500,000, the losses are 5,930,000, while the losses limited to \$20,000 are 5,050,000. The ILF is therefore $\frac{5930000}{5050000} = 1.17425742574$.

(b) The incremental method.

We calculate the following incremental ILFs:

Limit Increase	ILF
20,000–50,000	$\frac{41750000}{35690000} = 1.16979546091$
50,000–100,000	$\frac{36200000}{33740000} = 1.072910492$
100,000–500,000	$\frac{5930000}{5500000} = 1.07818181818$

The ILF from 20,000 to 500,000 is therefore $1.16979546091 \times 1.072910492 \times 1.07818181818 = 1.35321071516$.

2. *For a certain line of insurance, the loss amount per claim follows an exponential distribution with mean θ. If the policy has a deductible per loss set at 0.5θ and a policy limit set at 4θ, by how much will the expected payment per loss increase if there is inflation of 4%?*

For the current loss distribution, the expected payment per loss is

$$\begin{aligned}\int_{0.5\theta}^{4\theta} e^{-\frac{t}{\theta}}\,dt &= \left[-\theta e^{-\frac{t}{\theta}}\right]_{0.5\theta}^{4\theta}\\ &= \left(e^{-0.5} - e^{-4}\right)\theta\\ &= 0.5882215\theta\end{aligned}$$

After the inflation, the expected payment per loss is

$$\begin{aligned}\int_{0.5\theta}^{4\theta} e^{-\frac{t}{1.04\theta}}\,dt &= \left[-1.04\theta e^{-\frac{t}{1.04\theta}}\right]_{0.5\theta}^{4\theta}\\ &= 1.04\left(e^{-\frac{0.5}{1.04}} - e^{-\frac{4}{1.04}}\right)\theta\\ &= 0.62082237\theta\end{aligned}$$

The percentage increase in expected payment per loss is therefore $\frac{0.6208237}{0.5882150} - 1 = 5.543664\%$.

3. *An insurance company charges a risk charge equal to the square of the average loss amount, divided by 10,000. It has the following data on a set of 600 claims from policies with limit $500,000.*

Losses Limited to	*20,000*	*50,000*	*100,000*	*500,000*
Total claimed	*4,050,000*	*5,340,000*	*5,500,000*	*5,930,000*

Calculate the ILF from $100,000 to $500,000.

For the losses limited to $100,000, the expected loss amount is $\frac{5500000}{600} = 9166.66666667$. The risk charge is therefore $\frac{9166.66666667^2}{10000} = 8402.77777778$. For losses limited to $500,000, the expected loss amount is $\frac{5930000}{600} = 9883.33333333$, and the risk charge is $\frac{9883.33333333^2}{10000} = 9768.02777777$. The ILF is therefore $\frac{9883.33333333+9768.02777777}{9166.66666667+8402.77777778} = 1.11849644268$.

Standard Questions

4. *An insurer calculates the ILF from $1,000,000 to $2,000,000 on a particular policy is 1.074. The average loss per unit of exposure with the policy limit of $1,000,000 is $664. The insurer's premium also includes a risk charge equal to the square of the expected loss divided by 2,000. A reinsurer is willing to provide excess-of-loss reinsurance of $1,000,000 over $1,000,000 (that is, the attachment point is $1,000,000 and the limit on the reinsurer's payment is $1,000,000) for a premium of $58. Calculate the premium the insurance company should charge for a policy with limit $2,000,000*

(a) If they do not buy the excess-of-loss reinsurance

If the insurer does not buy the excess-of-loss reinsurance, the expected loss per unit of exposure is $664 \times 1.074 = 713.136$. The risk charge is $\frac{713.136^2}{2000} = 254.281477248$, so the total premium is $713.136+254.281477248 = \$967.42$.

(b) If they buy excess-of-loss reinsurance.

If they buy excess-of-loss reinsurance, the expected loss per unit of exposure is 664, and the risk charge is $\frac{664^2}{2000} = 220.448$, so the total premium is $664 + 220.448 + 58 = \$942.45$.

5. *An insurer computes the following trend factors for different policy limits:*

Policy Limit	*$50,000*	*$100,000*	*$500,000*	*$1,000,000*	*none*
Trend factor	*1.03*	*1.05*	*1.055*	*1.059*	*1.06*

The insurance company buys excess-of-loss reinsurance of $500,000 over $500,000 on its policies with policy limit $1,000,000. The loading on this reinsurance is 25%. The reinsurance premium is currently 5% of the insurer's expected loss payments. Calculate the reinsurance premium as a percentage of insurer's expected loss payments after applying the trend factors.

Let x be the expected aggregate payments by the insurer for policies with limit $500,000, and let l be the ILF from $500,000 to $1,000,000. Then the reinsurer's expected payment on the excess-of-loss coverage is $x(l-1)$. The reinsurance premium is therefore $1.25x(l-1)$, and we are told that this is 5% of x. That is

$$\begin{aligned}1.25x(l-1) &= 0.05x\\ 1.25(l-1) &= 0.05\\ l-1 &= 0.04\\ l &= 1.04\end{aligned}$$

After the trend factors are applied, the insurer's expected loss payments are $1.055x$, while the total expected loss benefits paid to the policy holder are $1.059lx = 1.059 \times 1.04x = 1.10136x$. The reinsurer's expected payment is therefore $1.10136x - 1.055x = 0.04636x$, and the reinsurer's premium is $1.25 \times 0.04636x = 0.05795x$. The percentage of insurer's expected loss payments is therefore $\frac{0.05795}{1.055} = 5.49289\%$.