ACSC/STAT 4703, Actuarial Models II
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Homework Sheet 5
Model Solutions

Basic Questions

1. *An insurance company sets the book pure premium for its workers' compensation insurance at \$732. The expected process variance is 54,906 and the variance of hypothetical means is 36,040. If a company has aggregate claims of \$232,700 on policies covering a total of 84 employees, calculate the credibility premium for this company's next year's insurance using the Bühlmann model.*

   The credibility is $Z = \frac{84}{84+\frac{54906}{36040}} = 0.98218648228$. The new premium is therefore $0.98218648228 \times \frac{232700}{84} + 0.01781351772 \times 732 = \$2,733.93$.

2. *An insurance company has the following data on a fire insurance policy for a company.*

Year	*1*	*2*	*3*	*4*	*5*
Exposure	*34*	*42*	*58*	*49*	*54*
Aggregate claims	*\$34,320*	*\$29,140*	*\$47,030*	*\$42,200*	*\$43,830*

   The book premium is \$760 per unit of exposure. The variance of hypothetical means per unit of exposure is 409,000. The expected process variance per unit of exposure is 8,048,600. Using a Bühlmann-Straub model, calculate the credibility premium for Year 6 if the company has 57 units of exposure.

   The company has made aggregate claims of \$196,520 on 237 units of exposure. The credibility of 237 units of exposure is $Z = \frac{237}{237+\frac{8048600}{409000}} = 0.923333231728$. The credibility premium is therefore

$$0.923333231728 \times \frac{196520}{237} + 0.076666768272 \times 760 = \$823.89$$

3. *An insurance company has the following previous data on aggregate claims:*

Policyholder	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Mean*	*Variance*
1	*412.83*	*1199.65*	*0.00*	*22.66*	*194.77*	*365.982*	*244580.86*
2	*0.00*	*1416.52*	*2239.52*	*0.00*	*96.68*	*750.544*	*1053686.07*
3	*0.00*	*0.00*	*167.69*	*0.00*	*563.23*	*146.184*	*59624.79*
4	*0.00*	*275.90*	*670.79*	*0.00*	*0.00*	*189.338*	*86708.91*

   Calculate the Bühlmann credibility premium for each policyholder in Year 6.

We need to estimate the EPV and VHM. We first estimate the EPV as the average of the observed variances:

$$\frac{1}{4}(244580.86 + 1053686.07 + 59624.79 + 86708.91) = 361150.1575$$

The mean aggregate claim amount for all 4 policyholders is

$$\frac{1}{4}(365.982 + 750.544 + 146.184 + 189.338) = 363.012$$

The variance of observed means is therefore

$$\frac{1}{3}((365.982-363.012)^2+(750.544-363.012)^2+(146.184-363.012)^2+(189.338-363.012)^2) = 75788.9705946$$

The variance due to process variance is $\frac{361150.1575}{5} = 72230.0315$, so the estimated VHM is $75788.9705946 - 72230.0315 = 3558.9390946$.

The credibility of 5 years of experience is therefore $Z = \frac{5}{5+\frac{361150.1575}{3558.9390946}} = 0.0469585358749$. This gives the following premiums in Year 6:

Policyholder	Premium
1	$0.0469585358749 \times 365.982 + 0.953041464125 \times 363.012 = \$363.15$
2	$0.0469585358749 \times 750.544 + 0.953041464125 \times 363.012 = \$381.21$
3	$0.0469585358749 \times 146.184 + 0.953041464125 \times 363.012 = \$352.83$
4	$0.0469585358749 \times 189.338 + 0.953041464125 \times 363.012 = \$354.86$

4. *Over a five-year period, an insurance company observes the following numbers of claims:*

No. of claims	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Frequency	*3163*	*3103*	*1896*	*1019*	*479*	*206*	*78*	*36*	*13*	*6*	*1*

Assuming the number of claims made by an individual in a year follows a Poisson distribution, calculate the credibility estimate for the expected claim frequency in the following year, of an individual who has made a total of 1 claim in the past 3 years. [Note that this is a different length of history from the individuals in the dataset.]

The average number of claims is

$$\frac{1}{10000}(3163 \times 0 + 3103 \times 1 + 1896 \times 2 + 1019 \times 3 + 479 \times 4 + 206 \times 5 \\ +78 \times 6 + 36 \times 7 + 13 \times 8 + 6 \times 9 + 1 \times 10) = 1.3786$$

Since the Poisson distribution has equal mean and variance, this means the EPV is 1.3786. The variance of the number of claims is

$$\frac{1}{9999}\big(3163 \times (0 - 1.3786)^2 + 3103 \times (1 - 1.3786)^2 + 1896 \times (2 - 1.3786)^2 + 1019 \times (3 - 1.3786)^2 \\ +479 \times (4 - 1.3786)^2 + 206 \times (5 - 1.3786)^2 + 78 \times (6 - 1.3786)^2 + 36 \times (7 - 1.3786)^2 \\ +13 \times (8 - 1.3786)^2 + 6 \times (9 - 1.3786)^2 + 1 \times (10 - 1.3786)^2\big) = 1.96585862588$$

This means that the VHM for number of claims in a five-year period is $1.96585862588 - 1.3786 = 0.58725862588$ and the credibility of five years of experience is

$$Z = \frac{1}{1 + \frac{1.3786}{0.58725862588}} = 0.298728819128$$

The credibility of three years of experience is therefore

$$Z = \frac{0.6}{0.6 + \frac{1.3786}{0.58725862588}} = 0.203561120767$$

For an individual who has made 1 claim in the past 3 years, the expected number of claims in the next year is

$$0.203561120767 \times \frac{1}{3} + 0.796438879233 \times \frac{1.3786}{5} = 0.287447834704$$

Standard Questions

5. *Aggregate claims for a given individual policy are modelled as following a gamma distribution with $\theta = 400$. The first 5 years of experience on this policy are:*

Policyholder	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Mean*	*Variance*
1	*839*	*427*	*36*	*38*	*466*	*361.2*	*113,454.7*
2	*256*	*1,276*	*496*	*903*	*564*	*699.0*	*157,557.0*
3	*146*	*112*	*682*	*557*	*1,022*	*503.8*	*146,161.2*
4	*524*	*1,856*	*2,783*	*1,438*	*652*	*1,450.6*	*859,036.8*

(a) Estimate the EPV and VHM.

Recall that the mean of a gamma distribution is $\alpha\theta$ while the variance is $\alpha\theta^2$. Therefore, using the method of moments, the variance is θ times the mean. The mean aggregate claim is

$$\frac{361.2 + 699.0 + 503.8 + 1450.6}{4} = 753.65$$

. The EPV is therefore $400 \times 753.65 = 301460$. The variance of observed means is

$$\frac{(361.2 - 753.65)^2 + (699.0 - 753.65)^2 + (503.8 - 753.65)^2 + (1450.6 - 753.65)^2}{3} = 235055.983333$$

The part of this due to process variance is $\frac{301460}{5} = 60292$, so the VHM is $235055.983333 - 60292 = 174763.983333$.

(b) Calculate the credibility premium for policyholder 4 in the next year.

The credibility of 5 years of experience is therefore

$$Z = \frac{5}{5 + \frac{301460}{174763.983333}} = 0.743499403227$$

The credibility premium for policyholder 4 is therefore

$$0.743499403227 \times 1450.6 + 0.256500596773 \times 753.65 = \$1271.83$$

6. *Claim frequency in a year for an individual follows a Poisson with parameter Λt where Λ is the individual's risk factor and t is the individual's exposure in that year. An insurance company collects the following data:*

	Year 1		*Year 2*		*Year 3*	
Policyholder	*Exp*	*claims*	*Exp*	*claims*	*Exp*	*claims*
1	*382*	*1*	*334*	*2*	*376*	*0*
2	*670*	*0*	*676*	*2*	*504*	*0*
3	*715*	*3*	*665*	*2*	*476*	*2*
4	*792*	*0*	*813*	*1*	*619*	*2*

In Year 4, policyholder 1 has 401 units of exposure. Calculate the credibility estimate for claim frequency for policyholder 1.

Individual 1 has 3 claims from 1092 units of exposure. Individual 2 has 2 claims from 1850 units of exposure. Individual 3 has 7 claims from 1856 units of exposure. Individual 4 has 3 claims from 2224 units of exposure. The overall average is therefore 15 claims from 7022 units of exposure or $\frac{15}{7022} = 0.002136143549$. This is the average value of Λ. Since the mean and variance for the Poisson distribution are equal, this is the EPV per unit of exposure. We now calculate

$$1092\left(\frac{3}{1092} - \frac{15}{7022}\right)^2 + 1850\left(\frac{2}{1850} - \frac{15}{7022}\right)^2 + 1856\left(\frac{7}{1856} - \frac{15}{7022}\right)^2 + 2224\left(\frac{3}{2224} - \frac{15}{7022}\right)^2 = 0.0088$$

The variance due to EPV is $4 \times 0.002136143549 = 0.008544574196$. The estimated VHM is therefore

$$\frac{0.00880939183012 - 0.008544574196}{3} = 0.0000882725447067$$

The credibility of 1092 units of exposure is therefore

$$Z = \frac{1092}{1092 + \frac{0.002136143549}{0.0000882725447067}} = 0.978319814263$$

The credibility estimate for the number of claims is therefore

$$401\left(0.978319814263 \times \frac{3}{1092} + 0.021680185737 \times \frac{15}{7022}\right) = 1.09633551831$$